Punjab University Journal of Mathematics (2023),55(7-8),327-342
https://doi.org/10.52280/pujm.2023.55(7-8)05

SH Wave Propagation in Dry Sandy Media Overlying an Orthotropic Elastic Layer and A Pre-Stressed Dry Sandy Half-Space

Dinesh Kumar Madan*, Naveen Kumar and Ritu Goyal
Mathematics Department, C. B. L. University, Bhiwani-127021(India)
*Corresponding author: dineshmadan@cblu.ac.in; *dk_madaan@rediffmail.com

Received: 06 April, 2022 / Accepted: 14 September, 2023 / Published online: 10 October, 2023

Abstract
SH wave propagation considering a model comprised of a dry sandy layer, an orthotropic elastic layer and a prestressed dry sandy half-space has been investigated in present problem with dislocation- like imperfectness considered at medium I and medium II's interface. Expressions for displacement components for layers and half-space are obtained by using variable separation method. Appropriate boundary conditions are used to derive dispersion equation and is used for graphical analysis. Numerical computations are used to depict the several characteristics' influence on SH wave propagation, such as sandiness, imperfection and pre-stress. Obtained results regarding propagation pattern of SH-waves for the considered geometry due to various parameters serves as the utmost features of this study, having possible applications in material sciences, soil dynamics and geophysics.
Keywords: SH wave, dry sandy, orthotropic elastic, imperfect, dispersion, pre-stress.

1. Introduction

Seismic waves cause physical damage due to earthquakes. So, the theoretical studies regarding seismic waves propagation in multilayered elastic media are critical for predicting seismic behavior in geophysics, soil, mechanics, earthquake science, structural engineering and many other fields.

At each level, the Earth's surface is thought to be more sandy than elastic. A sandy layer is made up of sand particles which do not absorb water vapour or moisture. The sandy layer has much low resistance due to slippage of granules particles. So, surface waves propagation involving sandy layers have been a topic of great interest in recent times to various researchers and seismologists. Weiskopf [24] investigated the mechanics of dry sandy soil introducing the term "sandy parameter". Some research studies regarding sandy media have been carried out by Paul [21], Tomar and Kaur [23] and Chattaraj [3]. Pal and Ghorai [18] investigated love wave propagation considering anisotropic porous half-space with gravity effects and pre-stressed sandy layer, observing gravity, initial stress and sandiness parameter effects. While Kumhar et al. [13] investigated Love wave propagation considering pre-stressed viscoelastic semi-infinite media and two sandy layers, Gupta and Ahmed [7] examined Love wave propagation using a model composed of sandy layer, fiber reinforced material layer and porous half-space. In order to account for the impacts of pre-stresses and sandy characteristics, Gupta et al. [8] explored SH-waves' transmission and reflection in a pre-stressed sandy stratum bounded by two initially stressed orthotropic semi-infinite media.

Although ideal interfaces have been considered in studies of seismic wave propagation in layered medium, the Earth's layers do not perfectly adhere to one another. The propagation pattern of seismic waves can be greatly affected by these contact circumstances. The results of these interfacial circumstances have been discussed by several researchers. Hua et al. [9] investigated the impacts of imperfectness parameter on Love wave propagation in a geometry made up of layered graded composite structures. In order to account for a viscoelastic layer covering a pair-stressed substrate with an imperfect interface and to observe the impacts of imperfectness, heterogeneity, friction, and imperfectness parameter, Sharma and Kumar [22] constructed a dispersion equation for the propagation of SH waves. In their study on shear wave propagation, Kumar et al. [13] took into account a micropolar elastic half-space and an initially strained piezoelectric layer with imperfect interface. Deep and Verma [4] investigated the effects of sandiness, imperfection, and parameter on the propagation of the Love wave in isotropic elastic half-space and dry sandy media. Kumar and Madan [12] investigated the effects of dry sand and the imperfectness parameter on the propagation of the Love wave in a layer of sand particles on top of an orthotropic semi-infinite medium.

Temperature variations, creeping, gravity variations and quenching process can be reasons for the generation of initial stresses. Earth is also a layered, highly elastic, initially stressed medium. So, studies regarding influences of initial stresses are very important to seismologists. Dey et al. [5] studied effects of initial stress and gravity parameter on torsional surface wave propagation considering sandy media. Ahmed and Abo-Dahab [1] examined Love wave propagation considering a granular layer of orthotropic media with effects of initial stress lying above granular half-space. Kundu et al. [14] observed effects of initial stresses on Love wave propagation considering orthotropic half-space and initially stressed fibre reinforced layer under. Madan et al. [16] investigated Rayleigh waves propagating in orthotropic elastic media under the initial stress effect observing the effects of rigidity and initial stresses. Various methods such as BEM were used by researchers ([19], [20], [17]) such as to solve the seismic wave propagation problem in linear spaces.

Keeping above theories and facts in mind, shear wave propagation in multi-layered sandy media with imperfectness effects remains yet to be unexamined. An attempt is made to investigate propagation of shear wave considering a dry sandy layer with a model comprised of sandy layer, orthotropic elastic layer and pre-stressed dry sandy half-space with dislocation like imperfectness considered at the sandy and orthotropic layer's interface. Displacement components obtained by using variable separation method. Suitable boundary conditions are used to derive the frequency equation for shear wave propagation. Special cases for absence of sandy layer or orthotropic layer have also been discussed and obtained frequency equation coincides with the already obtained results discussed as a special case. Through graphical analysis using MATLAB software, the significant effects of various parameters are shown for frequency and time domains on the SH wave propagation behavior.

2. Formulation and Basic Assumptions

Consider the SH wave propagating in a model comprised of a dry sandy layer (Medium 1), M_1 $: -h \leq z \leq 0$, and orthotropic elastic layer (Medium 2), M_2 $: 0 \leq z \leq +H$, over lying a pre-stressed sandy half-space (Medium 3), M_3 $: +H \leq z \leq \infty$. We have considered the Cartesian co-ordinate system (xyz system) for the problem and as our problem is anti-plane strain problem with origin O is located at sandy layer (M_1) and orthotropic elastic layer (M_2) interface and z-axis vertically downward and shear wave is assumed propagating along x-axis direction in dry sandy layer. Interface of sandy and orthotropic layer is assumed to be imperfect (dislocation-like imperfection).

SH-type Surface Wave Conditions:

If (u_i, v_i, w_i), i = 1, 2, 3 denotes the components of displacement for dry sandy layer, orthotropic elastic layer and pre-stressed sandy half-space respectively, then for SH-wave propagation in xz plane we may have:

$$\text{for i = 1, 2, 3;} \;\; u_i = 0 = w_i; \;\; v_i = v_i(x, z, t) \tag{1}$$

as our problem is anti-plane strain (-xz) problem.

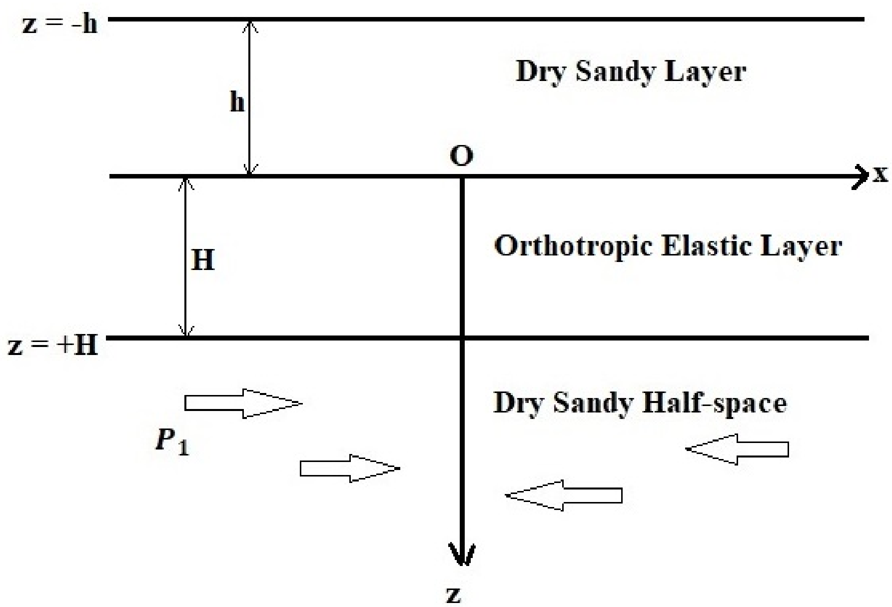

FIGURE 1. Geometry of the problem

Solution for Dry Sandy Layer

For SH wave propagation in sandy layer, equation of motion with zero external forces is given by Tomar and Kaur [23] as

$$\frac{\partial \sigma_{xy}^{(1)}}{\partial x} + \frac{\partial \sigma_{yz}^{(1)}}{\partial z} = \rho_1 \frac{\partial^2 v_1}{\partial t^2} \tag{2}$$

$$\text{with} \quad \sigma_{xy}^{(1)} = \frac{\mu_1}{\eta_1}\frac{\partial v_1}{\partial x}; \sigma_{yz}^{(1)} = \frac{\mu_1}{\eta_1}\frac{\partial v_1}{\partial z} \tag{3}$$

where $\sigma_{xy}^{(1)}$ and $\sigma_{yz}^{(1)}$ denote stresses and η_1, μ_1 denote sandiness and rigidity factor and $\rho_1$denotes density for Medium-I.
Using Equation (2) and (3)

$$\frac{\mu_1}{\eta_1}\frac{\partial^2 v_1}{\partial x^2} + \frac{\mu_1}{\eta_1}\frac{\partial^2 v_1}{\partial z^2} = \rho_1 \frac{\partial^2 v_1}{\partial t^2} \tag{4}$$

Assuming solution of the equation (4) as:

$$v_1(x, z, t) = V_1(z)e^{ik(x-ct)}. \tag{5}$$

Using Equation (4) and (5)

$$V_1''(z) + k^2\xi_1^2 V_1(z) = 0 \tag{6}$$

$$\text{where} \quad \xi_1^2 = \left(\frac{c^2}{\beta_1^2} - 1\right); \quad \beta_1^2 = \frac{\mu_1}{\eta_1\rho_1} \tag{7}$$

The solution of Equation (6) can be derived as

$$V_1(z) = A\cos(k\xi_1 z) + B\sin(k\xi_1 z) \tag{8}$$

Using Equation (8) in Equation (7), we have displacement component for dry sandy layer as:

$$v_1(z) = (A\cos(k\xi_1 z) + B\sin(k\xi_1 z))e^{ik(x-ct)} \tag{9}$$

where A and B are arbitrary constants.

Solution for Orthotropic Elastic Layer

Equation of motion with zero external forces for propagation of SH wave in orthotropic elastic layer given by Biot [2]

$$\frac{\partial \sigma_{xy}^{(2)}}{\partial x} + \frac{\partial \sigma_{yz}^{(2)}}{\partial z} = \rho_2 \frac{\partial^2 v_2}{\partial t^2} \tag{10}$$

with stresses given as:

$$\sigma_{xy}^{(2)} = C_{66}\frac{\partial v_2}{\partial x}; \quad \sigma_{yz}^{(2)} = C_{44}\frac{\partial v_2}{\partial z} \tag{11}$$

Here, C_{44} and C_{66} denotes elastic constants and ρ_2 denotes density for the orthotropic material. Using Equation (10) and (11)

$$C_{66}\frac{\partial^2 v_2}{\partial x^2} + C_{44}\frac{\partial^2 v_2}{\partial z^2} = \rho_2 \frac{\partial^2 v_2}{\partial t^2} \tag{12}$$

Assuming solution of the Equation (12) as:

$$v_2(x,z,t) = V_2(z)e^{ik(x-ct)}. \tag{13}$$

Using Equation (13) and (12)

$$V_2''(z) + k^2\xi_2^2 V_2(z) = 0 \tag{14}$$

$$\text{where} \quad \xi_2^2 = \frac{C_{66}}{C_{44}}\left(\frac{c^2}{\beta_2^2} - 1\right); \quad \beta_2^2 = \frac{C_{66}}{\rho_2} \tag{15}$$

Solution of Equation (14) is derived as:

$$V_2(z) = (C\cos(k\xi_2 z) + D\sin(k\xi_2 z)) \tag{16}$$

Displacement component in the orthotropic layer can be written as:

$$v_2(z) = (C\cos(k\xi_2 z) + D\sin(k\xi_2 z))e^{ik(x-ct)} \tag{17}$$

with C and D as arbitrary constants.

Solution for Sandy Half-space

Under the initial stress (P_1) effects, equation of motion for propagation of shear wave is as (Kar et al. [10])

$$\frac{\partial\sigma_{xy}^{(3)}}{\partial x} + \frac{\partial\sigma_{yz}^{(3)}}{\partial z} - P_1\left(\frac{\partial\omega}{\partial x}\right) = \rho_3\frac{\partial^2 v_3}{\partial t^2} \tag{18}$$

$$\text{where} \quad \omega = \left(\frac{\partial u_3}{\partial y} - \frac{\partial v_3}{\partial x}\right)/2, \quad \sigma_{xy}^{(3)} = \frac{\mu_2}{\eta_2}\frac{\partial v_3}{\partial x}; \sigma_{yz}^{(3)} = \frac{\mu_2}{\eta_2}\frac{\partial v_3}{\partial z} \tag{19}$$

$\sigma_{xy}^{(3)}$, $\sigma_{yz}^{(3)}$ denote stresse components and ω, μ_2 and η_2 denotes initial stress, rotational component, rigidity and sandiness factor and ρ_3 denotes material's density for half-space. Using Equations (18) and (19), we have

$$\frac{\mu_2}{\eta_2}\frac{\partial^2 v_3}{\partial x^2} + \frac{\mu_2}{\eta_2}\frac{\partial^2 v_3}{\partial z^2} + \frac{P_1}{2}\frac{\partial^2 v_3}{\partial x^2} = \rho_3\frac{\partial^2 v_3}{\partial t^2} \tag{20}$$

Assuming solution of the (20) be:

$$v_3(x,z,t) = V_3(z)e^{ik(x-ct)}. \tag{21}$$

Using Equation (20) and (21), we have

$$V_3''(z) - k^2\xi_3^2 V_3(z) = 0 \tag{22}$$

$$\text{where} \quad \xi_3^2 = \left(1 - \frac{c^2}{\beta_3^2} - P\right); \quad \beta_3^2 = \frac{\mu_2}{\eta_2\rho_3} \text{ and } P = \frac{P_1\eta_2}{2\mu_2} \tag{23}$$

Solution of Equation (22) can be obtained as:

$$V_3(z) = Ee^{k\xi_3 z} + Fe^{-k\xi_3 z} \tag{24}$$

From Equation (21): $v_3(z) = (Ee^{k\xi_3 z} + Fe^{-k\xi_3 z})e^{ik(x-ct)}$ (25)

with E and F as arbitrary constants. Also displacement must vanish as z $\to \infty$, so we must have E = 0. So, for sandy half-space, displacement component is:

$$v_3(x, z, t) = Fe^{-k\xi_3 z}e^{ik(x-ct)} \tag{26}$$

3. Boundary Conditions

Appropriate interfacial conditions for SH wave propagation for the geometry are given by

(1) Stress free upper surface of sandy layer (M_1), i.e., $\sigma_{yz}^{(1)} = 0$ at z = -h.
(2) Continuity of stresses for imperfect interface between sandy and orthotropic elastic layer. i.e. at z = 0, $\sigma_{yz}^{(1)} = \sigma_{yz}^{(2)}$.
(3) As the interface is dislocation like imperfect, so at z = 0, $v_2 = Gv_1$, where G denotes imperfectness parameter.
(4) Continuity of stresses and displacements for sandy half-space and orthotropic media, that is, at z = H, $\sigma_{yz}^{(2)} = \sigma_{yz}^{(3)}, v_2 = v_3$.

Applying these boundary conditions and using Equations (9), (17) and (26),

$$\begin{aligned} A\sin(k\xi_1 h) + B\cos(k\xi_1 h) &= 0 \\ B\tfrac{\mu_1}{\eta_1}\xi_1 - D\xi_2 C_{44} &= 0 \\ C - AG &= 0 \\ C\cos(k\xi_2 h) + D\sin(k\xi_2 h) - Fe^{-k\xi_3 H} &= 0 \\ -C\xi_2 C_{44}\sin(k\xi_2 h) + D\xi_2 C_{44}\cos(k\xi_2 h) + F\tfrac{\mu_2}{\eta_2}e^{-k\xi_3 H} &= 0 \end{aligned} \tag{27}$$

Equation (27) represents a homogeneous system of equation and for non-trivial solution, we must have:

$$\begin{vmatrix} \sin(k\xi_1 h) & \cos(k\xi_1 h) & 0 & 0 & 0 \\ 0 & \frac{\mu_1}{\eta_1}\xi_1 & 0 & \xi_2 C_{44} & 0 \\ -G & 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(k\xi_2 h) & \sin(k\xi_2 h) & -e^{-k\xi_3 H} \\ 0 & 0 & -\xi_2 C_{44}\sin(k\xi_2 h) & \xi_2 C_{44}\cos(k\xi_2 h) & \frac{\mu_2}{\eta_2}e^{-k\xi_3 H} \end{vmatrix} = 0 \tag{28}$$

Solving the determinant in Equation (28), we have:

$$\tan(kh\xi_1) = \frac{G\xi_2 C_{44}\left\{\frac{\mu_2\xi_3}{\eta_2} - \xi_2 C_{44}\tan(kh\xi_2)\right\}}{\frac{\mu_1}{\eta_1}\xi_1\left\{\frac{\mu_2\xi_3}{\eta_2}\tan(kH\xi_2) + \xi_2 C_{44}\right\}} \tag{29}$$

Equation (29) represents the dispersion relation for SH wave propagating in the considered geometry.

4. Special Cases

Case I: When M_2 is neglected, we have H = 0, then (29) becomes

$$\tan(kh\xi_1) = \frac{G\eta_1\mu_2\xi_3}{\eta_2\mu_1\xi_1} \tag{30}$$

Then Equation (30) represents frequency equation for dispersion of SH wave propagating in sandy layer lying over sandy half-space.

Case II: Further in Equation (30), if $\eta_1 = \eta_2 = 1$, G = 1, P = 0, then

$$tan\left(kh\sqrt{\frac{c^2}{\beta_1^2} - 1}\right) = \frac{\mu_2\sqrt{\left(1 - \frac{c_2}{\beta_3^2}\right)}}{\mu_1\sqrt{\left(\frac{c_2}{\beta_1^2} - 1\right)}} \tag{31}$$

Equation (31) represents classical SH wave equation (Love [15]), that validates the problem.

Case III: When upper dry sandy layer is neglected, we have h = 0, then Equation (29) becomes

$$\tan(kH\xi_2) = \frac{\mu_2\xi_3}{\eta_2 C_{44}\xi_2} \tag{32}$$

Equation (32) represents SH-wave's dispersion equation, propagating in orthotropic elastic layer lying over a half-space of dry sandy particles under pre-stresses.

Case IV: Further in Equation (32), if $\eta_2 = 1$ and $C_{44} = C_{66}$, then Equation (32) becomes

$$tan\left(kH\sqrt{\frac{c^2}{\beta_2^2} - 1}\right) = \frac{\mu_2\sqrt{\left(1 - \frac{c_2}{\beta_3^2}\right)}}{\mu_1\sqrt{\left(\frac{c_2}{\beta_2^2} - 1\right)}} \tag{33}$$

which is again classical equation for SH wave propagation, that validates the study.

5. Numerical Computation and Discussion

Dispersion equation (29) is used to compute and discuss the numerical results graphically to investigate the SH wave propagation characteristics for the considered model. Plotted graphs are used to observe phase velocity variation with respect to frequency and time-history domain, using the relationship ($t = 2\pi/kc$). Following data has been used for the purpose (Gubbins [6]):

i. For upper dry sandy layer: $\rho_1 = 3380kg/m^3,\ \mu_1 = 0.682 \times 10^{10} N/m^2,$
ii. For orthotropic elastic layer: $\rho_2 = 4500kg/m^3,\ C_{66} = 3.99 \times 10^{10} N/m^2$
$C_{44} = 5.82 \times 10^{10} N/m^2,$

iii. For dry sandy half-space: $\rho_3 = 3409kg/m^3,\ \mu_1 = 6.54 \times 10^{10} N/m^2.$

The imperfectness parameter effect is depicted in frequency and time domains, respectively, in Figs. 2 and 3. Variation in phase velocity against wave number and time is demonstrated by using 3 different values of imperfectness parameter and initial stress as 0.4 (when present) and sandiness parameters (η_1, η_2) as 1.5. From the plots, it can be seen that phase velocity increases with the increase in imperfectness parameter for the three cases for frequency-domain while phase velocity is significantly effected by the imperfect parameter for time-domain.

Fig. 4 and 5 show the sandiness factor (η_1) parameter effects. Fig. 4 and 5 (a), (b) are plotted for imperfect and perfect interface respectively. Variation in phase velocity is shown by using 3 different values of sandiness factor, with values of both initial stress and imperfectness (when present) taken as 0.3 and sandiness factor for half-space (η_2) is 1.5. Sandiness parameter increased the phase velocity for both frequency and time-domain in both perfect and imperfect cases except for perfect interface in time-domain, where it showed different behavior.

Fig. 6 depicts the effect of pre-stress parameter on SH waves' phase velocity. Fig. 6(a) and (b) are plotted for imperfect and perfect interface, respectively. Again, variation is shown using 3 different values of initial stress with values of sandiness factors(η_1, η_2) and imperfectness parameter(when present) as 1.5 and 0.3 respectively. Initial stress parameter decreased the SH-wave' phase velocity for imperfect or perfect interface, but in a slightly different manner.

Fig.7 and 8 show the thickness ratio effect in frequency and time-domain respectively. Values of sandiness parameters, initial stress, and imperfectness parameters were taken as 1.5, 0.3, and 0.4, respectively for plotting phase velocity to wave number. It is depicted that thickness ratio enhanced the phase velocity in frequency domain but has negligible effects in time-domain.

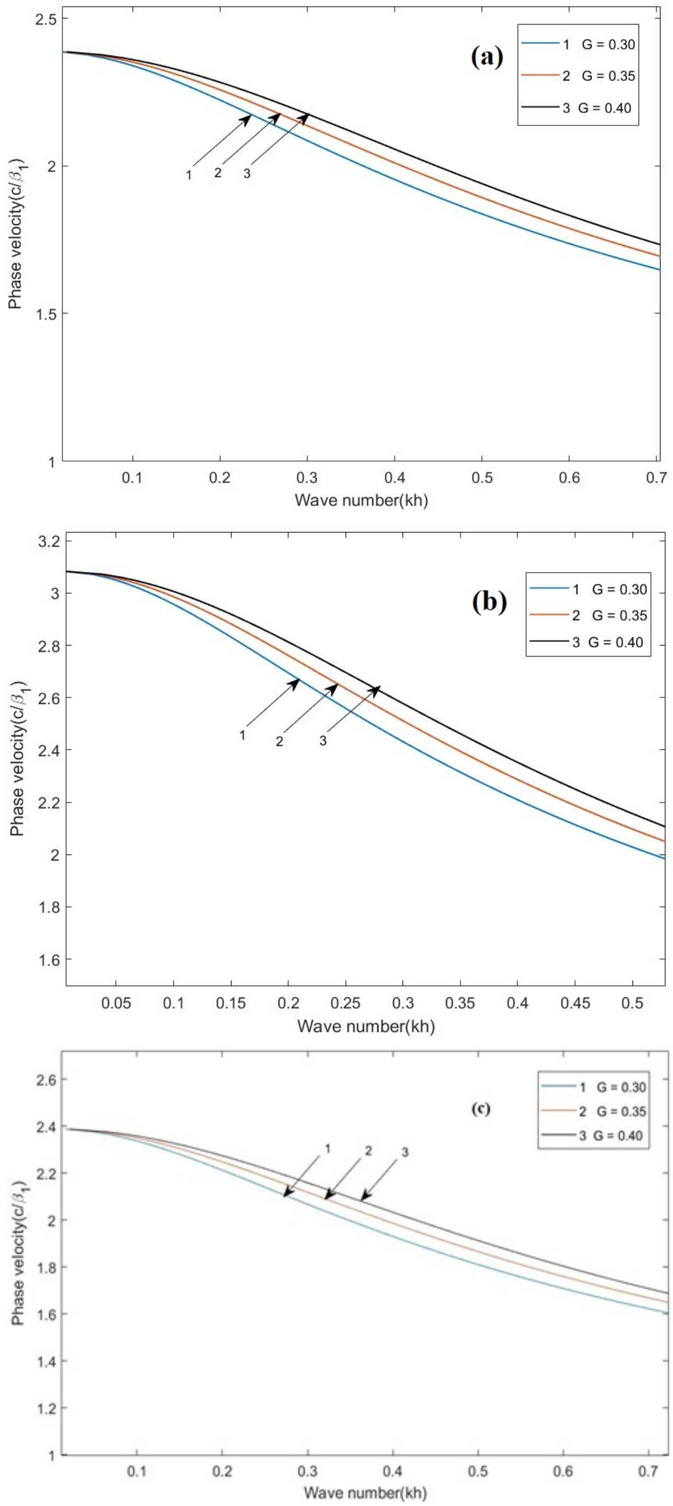

FIGURE 2. Imperfectness parameter effects in wave number domain in (a) initial stress presence (b) initial stress absence (c) for $\eta_1, \eta_2 = 1$, i.e. when dry sandy layers become isotropic

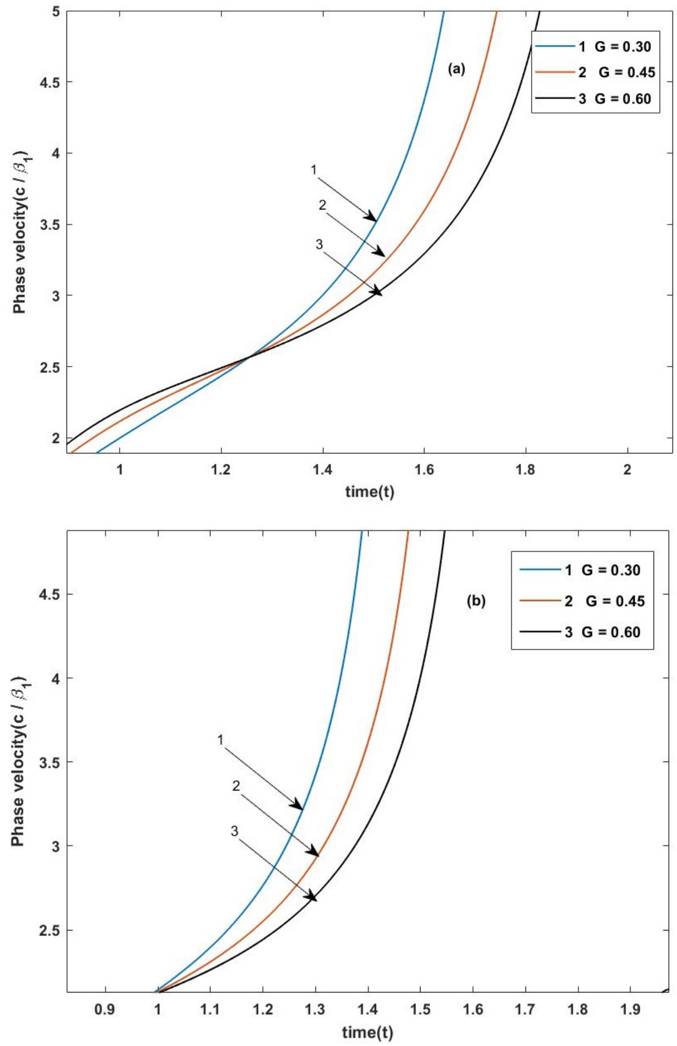

FIGURE 3. Imperfectness parameter effect in time-domain for (a) sandy media (b) for $\eta_1, \eta_2 = 1$, i.e. when dry sandy layers become isotropic

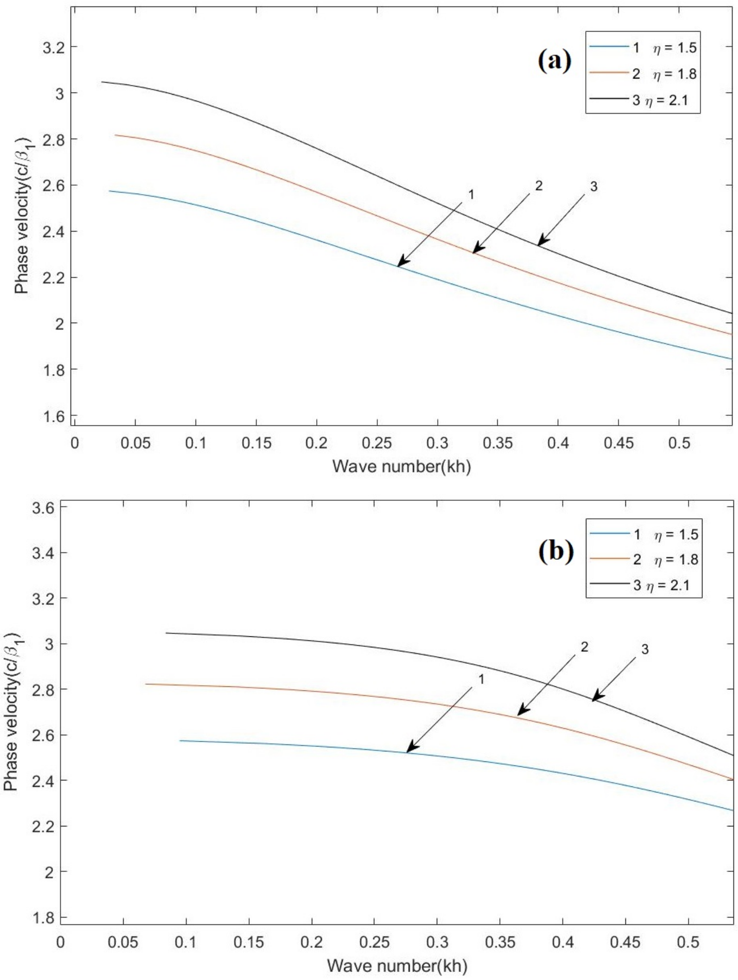

FIGURE 4. Sandiness parameter effects in wave number domain for (a) imperfect interface (b) perfect interface

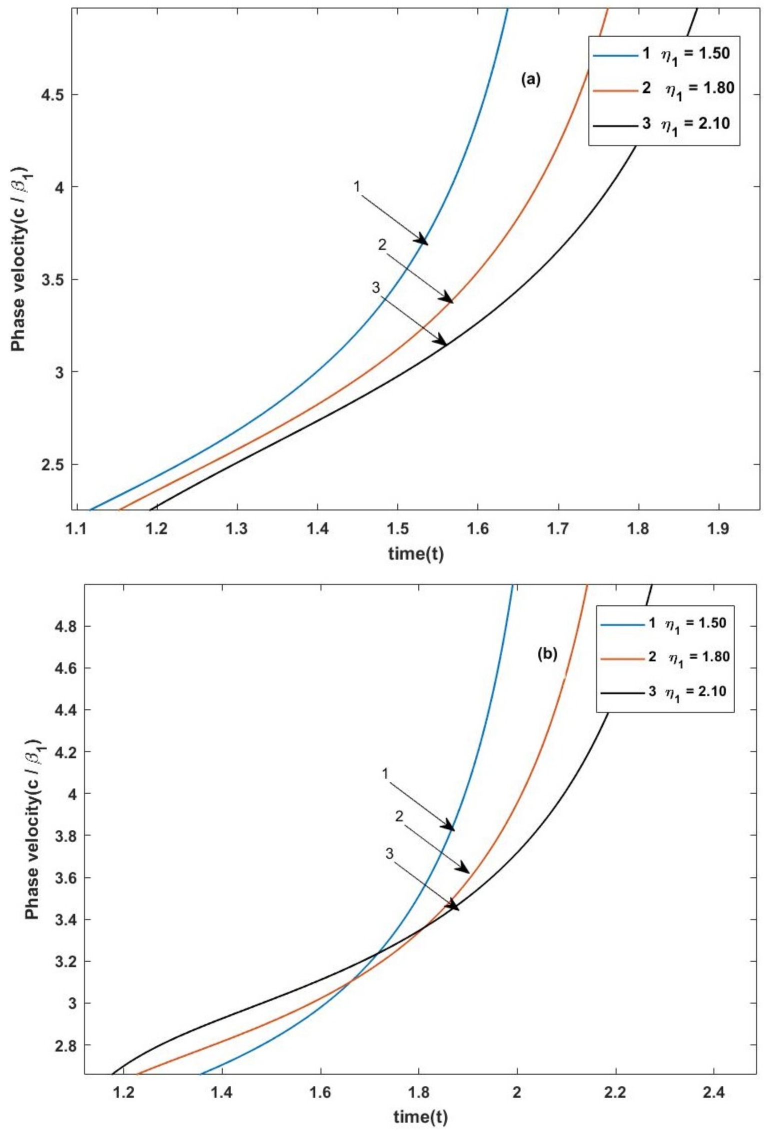

FIGURE 5. Sandiness parameter effects in time-domain for (a) imperfect interface (b) perfect interface

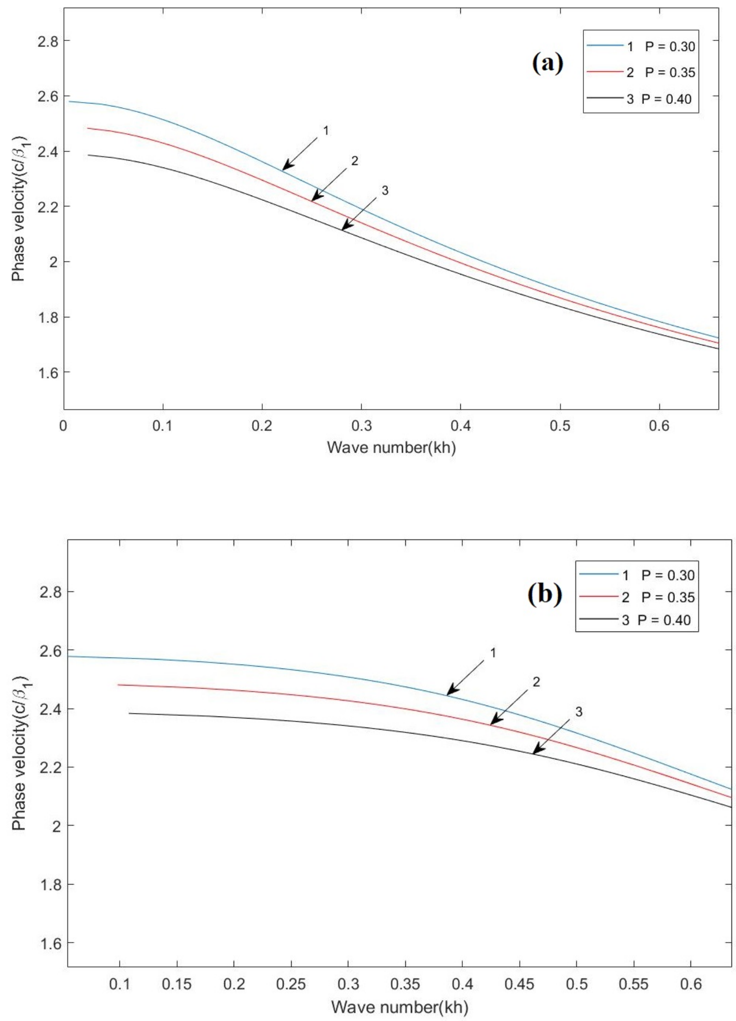

FIGURE 6. Initial stress effect in wave number domain for (a) imperfect interface (b) perfect interface.

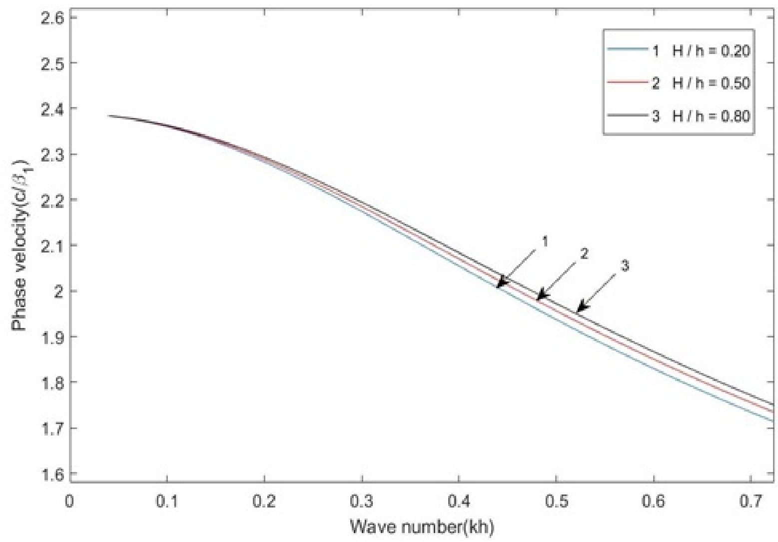

FIGURE 7. Thickness ratio effect in wave number domain

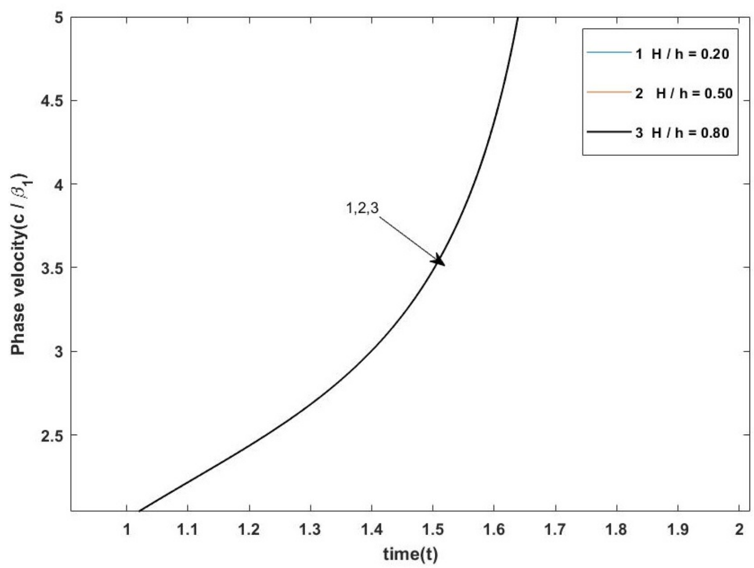

FIGURE 8. Thickness ratio effect on phase velocity(c/β_1) in time-domain

6. Conclusion

SH wave propagation in a geometry consisting of a sandy layer, an orthotropic elastic layer, and a pre-stressed sandy half-space is discussed. Interface between the layers is taken dislocation-type. Dispersion equation for SH wave is derived by using certain boundary conditions, and further, some particular cases are also discussed. Derived results are in agreement to the earlier obtained results (discussed in particular cases). Using MATLAB software, dispersion relation is analyzed graphically to imperfection, pre-stress and sandiness parameter effects. The conclusions of the present study can be summed up as

1. Imperfectness parameter has significantly effected SH waves' phase velocity. It increases with increase in imperfection in the frequency-domain but has the different behaviour in the time-history domain.

2. Sandiness parameter has increased the phase velocity of SH wave for both perfect and imperfect interfaces in frequency and time-history domains except for perfect interface in time-domain.

3.Initial stress parameter has adversely effected the phase velocity.

4. Thickness ratio parameter has also effected the phase velocity in frequency-domain but have negligible effects in time-domain.

On the basis of the present study, it can be observed that presence of imperfectness, initial stress, and sandy factor have significant effects on the propagation behavior of SH-waves. Results obtained in the present work may make significant contribution to studies involving seismic wave propagation regarding Earth's layered geometries or may find possible applications in geophysics or earthquake science.

Acknowledgement: The author(s) would like to acknowledge the reviewers for their kind suggestions to improve the paper.

REFERENCES

[1] S.M. Ahmed and S.M. Abo-Dahab, *Propagation of Love waves in an orthotropic Granular layer under initial stress overlying a semi-infinite Granular medium,* Journal of Vibration of Control **16**, (2010) 1845-1858.
[2] M.A. Biot, *Mechanics of Incremental Deformations*, John Wiley and Sons New York, (1965).
[3] R. Chattaraj, S.K. Samal and S. Debasis, *On torsional surface wave in dry sandy crust laid over an inhomogeneous half-space*, Meccanica **50**, No. 7 (2015) 1807-1816.
[4] S. Deep and V. Verma, *Love type waves in a dry sandy layer lying over an isotropic elastic half-space with imperfect interface*, J. of Physics : Conference Series **1531**, (2020).
[5] S. Dey, A.K. Gupta and S. Gupta, *On propagation of Love waves in dry sandy medium sandwiched between fiber reinforced layer and prestressed porous half-space* , J. Eng. Mech. **128**, No. 10 (2002) 1115-1118.
[6] D. Gubbins, *Seismology and Plate Tectonics*, London Cambridge University Press, (1990).
[7] S. Gupta and M. Ahmed, *On propagation of Love waves in dry sandy medium sandwiched between fiber reinforced layer and prestressed porous half-space,* Earthq Struc. **12**, No. 6 (2017) 619-628.
[8] S. Gupta, Smita, S. Pramanik and A. Pramanik, *Effect of dry sandiness parameter and initial stress on the scattering of plane SH wave,* Arab J Geosci. **11**, No. 9 (2018). DOI:10.1007/s12517-018-3536-0
[9] L. Hua, Y. Jia-ling and L. Kai-Xin, *Love waves in layered graded composite structures with imperfectly bonded interface,* Chin. J. Aeronaut. **20**, No. 3 (2006) 210-214.
[10] B.K. Kar, A.K. Pal and V.K. Kalyani, *Propagation of Love waves in an irregular dry sandy layer,* Acta GeophysPolonica **34**, (1986) 57–170.
[11] R. Kumar, K. Singh and D.S. Pathania, *Shear waves propagation in an initially stressed piezoelectric layer imperfectly bonded over a micropolar elastic half space,* Struct. Eng. Mech. **69**, No. 2 (2019) 121-129. DOI:10.1108/MMMS-08-2019-0143

[12] N. Kumar and D.K. Madan, *Propagation of Love waves in dry sandy medium laying over orthotropic semi infinite medium with imperfect interface,* Int. J. Grid and Distributed Computing **14**, No. 1 (2021) 2057-2064.

[13] R. Kumhar, S. Kundu, M. Maity and S. Gupta, *Study of Love-type wave vibrations in double sandy layers on half-space of viscoelastic: An analytic approach,* Multidiscipline Modelling in Materials and Struct. **16**, No. 4 (2020) 731-748.

[14] S. Kundu, S. Gupta and S. Manna, *Propagation of Love wave in fibre-reinforced medium lying over an initially stressed orthotropic half-space,* Int. J. for Numerical and Analytical Methods in Geomech. **38**, (2014) 1172- 1182.

[15] A.E.H. Love, *Mathematical theory of elasticity,* Cambridge University Press London, (1920).

[16] D.K. Madan, A. Rani and M. Punia, *A note on the effect of rigidity and initial stress on the propagation of Rayleigh waves in pre stressed orthotropic elastic layered medium,* Proceedings of the Ind. Nat. Sci. Acad. **87**, (2021) 487-498. https://doi.org/10.1007/s43538-021-00044-3

[17] S. Mojtabazadeh-Hasanlouei, M. Panji and M. Kamalian, *Scattering attenuation of transient SH-wave by an orthotropic Gaussian-shaped sedimentary basin,* Engineering Analysis with Boundary Elements **140**, (2022) 186-219.

[18] J. Pal and A.P. Ghorai, *Propagation of Love wave in sandy layer under initial stress above anisotropic porous half- space under gravity,* Transp. Porous Med. **109**, No. 2 (2014) 297-316.

[19] M. Panji, M. Kamalian, J.A. Marnani and M.K. Jafari, *Transient analysis of wave propagations problems by half-plane BEM,* Geophysical Journal International **194**, (2013) 1849-1865.

[20] M. Panji and S. Mojtabazadeh-Hasanlouei, *On subsurface box-shaped lined tunnel under incident SH-wave propagation,* Frontiers of Structural and Civil Engineering **15**, (2021) 948–960.

[21] M.K. Paul, *Propagation of Love waves in a dry sandy layer lying between two semi-infinite elastic media,* Acta Geophys Pol. **13**, No. 1 (1965) 1-7.

[22] V. Sharma and S. Kumar, *Dispersion of SH waves in a viscoelastic layer imperfectly bounded with a couple stress substrate,* J. Theor. App. Mech. Pol. **55**, No. 2 (2017) 535-546.

[23] S.K. Tomar and J. Kaur, *SH-waves at a corrugated interface between a dry sandy half-space and an anisotropic elastic half-space,* Acta Mechanica **190**, (2007) 1-28. DOI:10.1007/s00707-006-0423-7

[24] W.H. Weiskopf, *Stresses in soils under foundation,* J. Franklin Inst. **239**, (1945) 445-465.