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Optimizing decision making for sustainable solar thermal energy systems within an intuitionistic fuzzy environment

Muhammad Zafar Abbas
Department of Mathematics and Statistics,
University of Lahore, Lahore, Pakistan

*Rukhshanda Anjum
Department of Mathematics and Statistics,
University of Lahore, Lahore, Pakistan

Corresponding Author
rukhshanda.anjum@math.uol.edu.pk

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Abstract. In the context of addressing global energy needs and climate change, the transition to sustainable energy systems has become one of the most important objectives, and in this context. Solar-thermal energy systems are among the most important sources of renewable energy solutions. However, existing research on solar-thermal energy systems often falls short in effectively handling uncertainty and the credibility of expert evaluations, which are essential for ensuring reliable decision-making. Traditional methods still focus on deterministic or simplified models, neglecting the intricate and uncertain evaluations present in real-world situations. To address these shortcomings, this study employs intuitionistic fuzzy credibility numbers (IFCNs), which explicitly incorporate membership, non-membership, and credibility degrees, enabling a more comprehensive representation of uncertain and hesitant information. Based on IFCNs, we introduce a new, more effective method of handling credibility in aggregation named the generalized intuitionistic fuzzy credibility numbers weighted averaging (GIFCNWA) operator. Based on this operator, a multi-attribute decision-making algorithm is proposed that integrates attribute importance and reliability of evaluations. The proposed methodology is applied to a real-world decision-making problem in solar thermal energy storage by considering four options: nanostructured absorber coatings, advanced phase-change materials, high-temperature alloys, and adaptive control systems. The results identify nanostructured absorber coatings as the most optimal solution. A parametric sensitivity analysis is conducted, demonstrating the robustness and stability of the proposed operator under varying parameter conditions. A comparative analysis with existing decision-making techniques reveals the superiority and consistency of the proposed operator in delivering reliable results.

Key Words: Intuitionistic fuzzy set, Credibility numbers, Generalized intuitionistic fuzzy credibility numbers weighted averaging operator, Multi-attribute decision-making, Sustainable energy systems, Solar energy

1. Introduction

Due to the rising global energy demands, the shift to sustainable energy sources has become one of the primary concerns of the 21st century, driven by increasing energy needs, the depletion of fossil fuels, and the urgent need to mitigate climate change. Among the various renewable energy sources, solar thermal energy can capture solar radiation and convert it into heat energy with remarkable efficiency. This makes solar thermal energy a fundamental component of sustainable energy generation and storage systems. In recent years, extensive research has been conducted to advance thermal energy storage technologies, which play a crucial role in bridging the gap between the availability of solar energy and fluctuating demand. This enhances system efficiency and reliability by storing excess thermal energy for later use, thereby ensuring a continuous energy supply and improving economic feasibility. Many thermal energy storage systems, whether utilizing sensible heat, latent heat, or thermochemical heat, continue to show considerable promise in solar applications [9]. Their findings show that thermochemical systems can substantially reduce storage volume, while materials such as molten salts and graphite can be utilized as high-energy-density and thermally conductive materials to create effective energy storage systems.

Building upon recent advancements, research has also considered the combination of solar thermal systems with thermal storage into fossil fuel-powered plants, concentrating primarily on the techno-economic viability of such hybrid configurations [5]. This study highlights how combining a fuel plant with a 4-hour delay thermal energy storage system significantly enhances the system's performance, reduces fuel consumption, and provides substantial savings, underscoring the economic and environmental viability of solar-assisted thermal plants. In addition to these, the growing importance of concentrated solar power technologies in achieving sustainable energy targets has also been recognized [15]. These systems, capable of operating beyond daylight hours through efficient energy storage, have more benefits compared to traditional photovoltaic systems. The authors emphasized that continuous advancements in energy storage technologies, in conjunction with innovative designs and cost reductions, are necessary to make solar thermal systems competitive and scalable. The authors also stated that, despite all these innovative efforts, the system design and optimization of solar energy systems continue to outperform due to the uncertainty of the complex interdependencies between the performance parameters of the systems and their surrounding environment. Despite these efforts, decision-making under uncertainty remains a major challenge in the design and optimization of solar energy systems, primarily due to complex interdependencies among performance parameters and the variability of environmental factors.

Traditional optimization and decision-making approaches in solar energy research often rely on deterministic models, which fail to adequately capture the uncertainty, hesitation, and credibility of expert evaluations. These limitations can lead to the inappropriate prioritization of options and a less-than-ideal configuration of systems. To overcome these shortcomings, fuzzy set theory and its extensions have been widely employed to handle vagueness and ambiguity in decision-making. Within this context, among the various fuzzy set operations, fuzzy aggregation operators are particularly powerful in consolidating uncertain and conflicting data, thereby providing a flexible and reliable framework for multi-attribute decision-making (MADM) problems. Over the years, several intuitionistic fuzzy aggregation operators, such as the intuitionistic fuzzy weighted averaging and intuitionistic fuzzy ordered weighted averaging operators, have been developed to extend the conventional fuzzy logic framework by incorporating the degrees of membership, non-membership, and hesitation. These operators have been successfully applied across domains, including renewable energy planning, supplier selection, and risk assessment, offering a more flexible representation of uncertainty compared to classical fuzzy methods. These operators, although mathematically attractive, fail to address substantial flaws in real-world scenarios and empirical decision-making contexts. Most existing models assume uniform reliability of information and fail to distinguish between highly credible and less reliable expert evaluations, resulting in biased aggregation results. Moreover, they often struggle to effectively capture the multi-dimensional nature of uncertainty, particularly when hesitation and vagueness coexist with variability in credibility. The aggregation process in such operators tends to oversimplify uncertain information, neglecting the confidence level associated with each judgment. As a result, their outcomes may lack robustness and interpretability in dynamic environments, such as energy system optimization, where decision parameters are interdependent.

To address these critical limitations, this study introduces the GIFCNWA operator as a new robust aggregation tool, integrating IFCNs to incorporate the credibility and reliability of expert evaluations explicitly. This proposed operator still captures any hesitation and uncertainty, but now it also adjusts according to the credibility of the parameters, thereby improving the realism and accuracy of the outcome. Unlike traditional aggregation approaches that treat all expert opinions with equal weight, the GIFCNWA operator differentiates inputs based on their reliability and contextual significance, thereby reducing the impact of inconsistent or less credible information. Furthermore, the operator demonstrates mathematical stability and adaptability, enabling consistent performance across diverse decision-making environments. Its design accommodates the combination of qualitative and quantitative analyses, facilitating the integration of subjective evaluations with objective numerical data. From this, a MADM methodology is constructed to facilitate the proposed advanced aggregation tool's integrated evaluation of complicated systems in a straightforward and accountable manner. To demonstrate the applicability and effectiveness of the proposed approach, a real-world application of solar thermal energy storage systems is provided as a case study. By incorporating credibility-based intuitionistic fuzzy modeling and sophisticated aggregation, this study advances decision science in the context of optimizing renewable energy resources. The proposed GIFCNWA framework effectively addresses the demands of reliable and intelligent decision-making systems, thereby bridging the gap between theoretical advancements and practical applications within the domain of sustainable energy systems.

1.1. **Literature review.** Fuzzy sets have revolutionized the handling of uncertainty and vagueness in various fields [46], allowing elements to possess a degree of membership rather than being limited to binary inclusion. Since then, this foundational concept has evolved through numerous extensions and has been widely applied across scientific and engineering domains. Fuzzy set theory was a great leap beyond classical set theory, as it can capture more subtle real modeling of real phenomena characterized by fuzziness [14]. Extensions such as HyperSoft Sets, Super HyperSoft Sets, and Indeterminate Soft Sets have emerged to address complex data-driven applications, particularly in databases and real-world decision-making problems. Soft set extensions play a crucial role in enhancing the accuracy and efficiency of healthcare data analysis, as they facilitate better decision-making and policy formulation [19]. A fuzzy hypersoft set framework with a novel energy metric based on singular values has been proposed to quantify multi-sub-attribute uncertainty, achieving superior performance in machine learning driven decision-making applications [23]. A decision-making approach using an interval-valued neutrosophic soft set has been presented to ensure fair distribution of financial aid [39]. Building on this foundation, fuzzy environments have also been refined through approaches such as Fermatean fuzzy sets and linguistic variables, which have further strengthened decision-making in uncertain contexts [13]. Moreover, Fuzzy set theory has been combined with number theory to produce hybrid frameworks, such as linear Diophantine fuzzy sets, with applications to pharmacology, power, and finance [3]. Recent innovations, including new extension principles and α weak operations, have been introduced to overcome the limitation of the traditional fuzzy set methods, enabling fuzzy sets to tackle problems involving incomplete or inconsistent data [25].

Traditional fuzzy sets are extended into intuitionistic fuzzy sets [7], which account for both the membership and non-membership degrees of a set element, thereby guiding uncertainty more effectively. In recent years, intuitionistic fuzzy sets have been increasingly applied in various fields, particularly in natural decision-making and mathematical modeling. A more comprehensive framework for dealing with uncertainty, in contrast to traditional fuzzy sets [26], is built based on intuitionistic fuzzy sets, which include a membership degree, a non-membership degree, and a degree of indeterminacy. It is particularly well-suited for group decision-making problems where the pros and cons, as well as the uncertainty of several alternatives, are to be considered without pairwise comparison [31]. In multi-criteria group decision-making, determining criteria weights using triangular intuitionistic fuzzy numbers is proposed, which highlights the significance of consensus among experts and allows for hesitant judgments [22]. The correlation coefficients for intuitionistic fuzzy sets have been developed, and their work has improved the reliability of multi-criteria decision-making applications in various cases, such as medical diagnosis and clustering, demonstrating better performance than ordinary coefficients [8]. A bibliometric analysis of fuzzy research advancements in Pakistan [41] has been conducted highlighting the publication growth research trends in theoretical and applied decision-making domains.

Intuitionistic fuzzy methods are applied practically in portfolio decision models to achieve return targets while controlling risk using a differential evolution algorithm [48]. Although intuitionistic fuzzy sets possess properties that are useful for dealing with uncertainty, some researchers claim that they are too complex, making it difficult to apply the framework in practical applications where straightforward decision-making models are required.

To enhance reliability and interpretability in decision models, the concept of credibility numbers was introduced. Originally emerging from actuarial and statistical domains, credibility theory provides a mathematical basis for combining information from multiple sources and quantifying the reliability of estimations [12]. When extended into fuzzy systems, credibility measures enable ranking and comparison of fuzzy quantities, thereby enriching decision-making processes with an additional layer of trust evaluation [21]. This led to the development of IFCNs [34], which integrates membership, non-membership, and hesitation degrees along with their corresponding credibility measures. By quantifying both uncertainty and confidence simultaneously, IFCNs improve the reliability of fuzzy assessments and allow more trustworthy decision-making. Their usefulness has been demonstrated in fields such as railway train selection and portfolio decision-making. Furthermore, credibility entropy measures associated with IFCNs have enhanced pattern recognition and survey sampling precision [47]. In the context of electronic data interchange, Pakistan [37] shows how this approach reduces ambiguity and increases the validity of survey results. For statistical decision-making when information is uncertain, interval-valued intuitionistic fuzzy confidence intervals are developed. Many of the same intervals apply to other statistical measures, including population means and proportions, and may be viewed as fuzzy hypothesis tests. In the case of incomplete data, they provide a robust decision-making framework [24]. The credibility measure for intuitionistic fuzzy variables quantifies the likelihood of fuzzy events, providing a foundation for calculating expected values and entropy. This measure is crucial for transforming fuzzy inventory models into deterministic problems, which can then be solved using soft computing techniques [18, 35]. IFCNs are utilized to construct similarity measures and a multi-criteria decision-making method [45], particularly for the performance evaluation of industrial robots. To manage ambiguity in data, fuzzy credibility rough sets were introduced, which define new aggregation operators to foster decision-making in green supplier selection [44]. Despite their advantages, IFCNs can be computationally intensive and require domain-specific interpretation, highlighting the need for simplified yet powerful tools that preserve credibility without adding excessive complexity.

The study of aggregation operators remains central to fuzzy and intuitionistic fuzzy decision-making frameworks, as these operators determine how multiple fuzzy values are combined into a collective assessment. These operators integrate individual expert evaluations into a unified decision, making them essential for accurate, transparent, and interpretable outcomes. In linguistic interval-valued intuitionistic fuzzy environments, Dempster-Shafer evidence theory is integrated, and weighted averaging operators are developed that are invariant and persistent. This way, monotonicity is guaranteed, and paradoxes are avoided [33] in decision-making situations. Weighted additive and geometric elliptic intuitionistic fuzzy operators provide the means to address non-uniform point distributions.

In multi-criteria decision-making, these operators are useful for ranking alternatives based on their distance to an ideal alternative [38]. Various aggregation operators are therefore developed using new logarithmic operational laws for intuitionistic fuzzy sets. The confidence logarithmic intuitionistic fuzzy Einstein weighted geometric operator and other such operators present better effectiveness than prevailing methods [36]. In computer science and decision-making, the ordered weighted averaging operator has been introduced by Yager [16]. This enables flexible aggregation according to the weights assigned to multiple criteria, based on their varying degrees of importance. In MADM, Pythagorean fuzzy aggregation operators have become essential tools, especially when information is imprecise [2]. Improvements in the visualization of complex decision-making situations were made by using a three-dimensional balance model for bipolar aggregation operators[40]. Experts benefit from this model to understand the aggregation process, such as in the selection of energy technologies. It expanded the applicability of aggregation operators in dealing with indeterminate information by introducing interval-valued intuitionistic fuzzy hypersoft sets [49]. Such an approach has been successfully implemented for the material selection problem, demonstrating its ability to cope with the environment of multi-criteria group decision-making. Although, aggregation operators greatly improve the decision-making process but their complexity makes it difficult to interpret and fail to incorporate the credibility of expert opinions directly, which limits their practical effectiveness.

To address such challenges in decision-making under uncertainty, the MADM framework has emerged as a powerful tool that complements aggregation theory. By integrating fuzzy and intuitionistic fuzzy aggregation techniques, MADM enhances flexibility and robustness in decision-making processes across various domains. For example, the fuzzy MADM method has been proven helpful in the Indonesian property market to help individuals choose appropriate houses [20]. It is essential to recognize that, despite the structured approaches to complex decision-making provided by MADM, subjective judgments often pose challenges in practice, and there is a need for robust decision support systems to enable reliable and accurate results. A MADM approach using neutrosophic credibility numbers has been proposed, showing the approach's versatility in allocating renewable energy resources in a smart grid [1]. Engineering decision-making involves various methodologies and frameworks that combine data-driven methods, quantitative analysis, and stakeholder expectations. Data-driven decision-making is critical to engineering management, and an advanced analytics capability allows one to achieve such efficiency and innovation [4]. Historical data is utilized for ranking alternatives in the decision-making methods [27]. Engineering Management involves utilizing quantitative analysis, as well as an understanding of the technical requirements and resource availability of the project [17]. The engineering design process must be multidisciplinary, incorporating stakeholder needs through the use of value-driven design and Game Theory [43]. Using this process, which formalizes a decision-making process, conflicts as well as synergies that may exist among stakeholders are assessed, and an optimal design solution is achieved. MADM has been used extensively in evaluating the conventional and unconventional machining processes to select the most efficient method based on one or many other performance criteria [10]. A Linear Diophantine Fuzzy Z-number–based decision-making approach has been proposed to handle uncertainty in textile engineering [30]. MADM, in its versatility, can be used for structuring decision processes and improving fairness in practical applications, such as civil servant recruitment [28]. In this case study, aggregation operators play a key role in determining preferences and criterion weights in MADM problems [32]. The aggregation techniques that engineering teams use to synthesize expert opinions [42] can suffer from inconsistencies that can debase decision quality. Due to its capability of integrating subjective judgments and managing uncertainty, MADM forms a robust framework for decision-making in mechanical engineering, with a significant impact on the reliability of decision outcomes. While improvements in aggregation techniques have been made, there is still considerable work to be done in developing reliable and consistent aggregation methods for real-world applications.

1.2. **Motivation for the research.** Modern decision-making faces significant challenges, particularly in addressing uncertainty, conflicting criteria, and incomplete data. Traditional methods, such as the weighted average method and simple additive weighting, often fall short in capturing the complexities of real-world problems. These methods treat all inputs as equally reliable, failing to account for variations in data credibility or expert confidence. Moreover, they oversimplify the intricate relationships between attributes, leading to generalized and often suboptimal solutions. For example, solar-thermal energy systems face significant challenges due to the inherent complexity of their operation and the interplay of various performance factors. Current approaches struggle to model the hesitation and uncertainty involved in determining these complex relationships. Driven by these gaps, we introduce them as a new framework that combines IFCN with the GIFCNWA operator. A structure of credibility degrees is proposed to address the shortcomings of traditional methods by modeling membership, non-membership, and credibility degrees. The GIFCNWA operator computes a precise aggregation of attributes and takes into account the reliability of each evaluation. We investigate redefining MADM, proposing a robust, scalable, and adaptable framework. Furthermore, this innovative model addresses the difficulties encountered by current methodologies, allowing decision-makers to determine the best solution in situations where determining the optimal solution is challenging and uncertain. This study contributes to the advancement of decision science by providing transformative solutions for real-world applications through this work.

1.3. **Research gaps and contributions.** Despite significant advancements in MADM methods, several critical research gaps remain unaddressed, particularly in the context of uncertainty and complexity inherent in real-world decision-making scenarios. However, traditional approaches often overlook the hesitation, vagueness, and varying degrees of credibility present in expert evaluations. As a result, they tend to oversimplify the interdependencies among attributes, leading to suboptimal outcomes that fail to reflect the detailed complexities of systems like solar-thermal energy. Beyond that, existing methodologies lack parameterization, which limits their capability to adapt naturally to changing inputs and varying conditions. To address these limitations, our research introduces a novel framework that integrates IFCNs with the GIFCNWA operator. By leveraging the flexibility and depth of IFCNs, this framework models membership, non-membership, and credibility degrees with precision, providing a comprehensive representation of uncertainty. The GIFCNWA operator enhances decision-making by aggregating these evaluations in a weighted and credibility-sensitive manner, ensuring balanced and reliable outcomes. The contributions of this research are both theoretical and practical in nature. Theoretically, it advances the field of decision science by introducing an innovative algorithm that bridges the gaps in existing methodologies. From a practical perspective, it constitutes a scalable and robust framework to address certain classes of complex real-world decision-making problems whose effectuation involves uncertainty and interdependent factors. This research makes significant contributions beyond the challenges in MADM, while also laying the foundation for future innovations in decision-making under uncertainty. The main objectives of this research are as follows:

- To propose an innovative aggregation method using the GIFCNWA operator that integrates attribute importance and evaluation reliability.
- To develop a robust MADM algorithm capable of handling uncertainty, hesitation, and vagueness in decision-making.
- To apply the proposed framework to a real-world problem of optimizing solar-thermal energy systems and demonstrate its effectiveness in addressing the challenges of performance and adaptability.
- To contribute a scalable and adaptable decision-making model applicable across diverse fields, including renewable energy, engineering, and economics.

1.4. **Structure of the manuscript.** The rest of the article is structured in the following way for better understanding and clarity:

In Section 2, the theoretical foundation is introduced, where key concepts and preliminaries about IFCNs are defined, which are crucial to understand the methodologies developed in the remainder. In Section 3, the GIFCNWA operator is proposed. In this section, we present the mathematical formulation of the operator, along with rigorous proofs of its central properties: idempotency, monotonicity, and boundedness. These properties guarantee the robustness and effectiveness of the operator. In Section 4, a MADM algorithm is proposed that integrates the GIFCNWA operator with IFCNs. The algorithm is designed to respond to uncertainty, hesitation, and interdependencies among attributes with a clear and replicable step-by-step explanation. Section 5 provides practical utility of the proposed framework through a detailed case study on solar-thermal energy systems. The algorithm is illustrated numerically in this section, and its robustness is validated through a sensitivity analysis. Managerial implications demonstrating the strategic value of this approach in real-world settings are presented in this section. Section 6 concludes this manuscript in the form of summarising the findings of this research, pointing to the contributions of this work, and suggesting future research.

2. Preliminaries.

The IFCNs represent a critical new approach to fuzzy logic as a solution to many decision-making processes for which uncertainty, vagueness, or lack of information are key elements. Extending the concept of intuitionistic fuzzy sets, IFCNs integrate three key elements: membership, non-membership, and a credibility degree to measure their reliability. Employing this dual-layer framework provides a complete uncertainty representation that clearly makes a distinction between inherent fuzziness and the degree of confidence in the data. IFCNs are very useful in capturing hesitation and verifying credibility, and are now applied in fields such as MADM, risk analysis, and resource optimization. IFCNs fill a gap between regular fuzzy systems and contribute to robust, informed, and reliable decision-making.

Definition 2.1. [34] Consider $\mathcal{X}$ as a universe set. An intuitionistic fuzzy credibility number set on $\mathcal{X}$ can be defined as follows:

$$\mathcal{P} = \{\langle \mathfrak{s}, \hat{\mu}(V,C)(\mathfrak{s}), \hat{\nu}(V,C)(\mathfrak{s})\rangle \mid \mathfrak{s} \in \mathcal{X}\},$$

where $\hat{\mu}(V,C)(\mathfrak{s}) : \mathcal{X} \to [0,1]$ and $\hat{\nu}(V,C)(\mathfrak{s}) : \mathcal{X} \to [0,1]$ are ordered pairs representing the membership and non-membership values along with their credibility values respectively. These values must satisfy the conditions $0 \leq \hat{\mu}_V(\mathfrak{s}) + \hat{\nu}_V(\mathfrak{s}) \leq 1$ and $0 \leq \hat{\mu}_C(\mathfrak{s}) + \hat{\nu}_C(\mathfrak{s}) \leq 1$. Simply, the element $\langle \mathfrak{s}, \hat{\mu}(V,C)(\mathfrak{s}), \hat{\nu}(V,C)(\mathfrak{s})\rangle$ in $\mathcal{P}$ is represented as $p = \langle \hat{\mu}(V,C), \hat{\nu}(V,C)\rangle = \langle(\hat{\mu}_V, \hat{\mu}_C), (\hat{\nu}_V, \hat{\nu}_C)\rangle$, which is termed as intuitionistic fuzzy credibility number.

Definition 2.2. [34] If $p = \langle \hat{\mu}_1(V,C), \hat{\nu}_1(V,C)\rangle = \langle(\hat{\mu}_{V1}, \hat{\mu}_{C1}), (\hat{\nu}_{V1}, \hat{\nu}_{C1})\rangle$ and $p_2 = \langle \hat{\mu}_2(V,C), \hat{\nu}_2(V,C)\rangle = \langle(\hat{\mu}_{V2}, \hat{\mu}_{C2}), (\hat{\nu}_{V2}, \hat{\nu}_{C2})\rangle$, then we give the following relations for $\eta > 0$.

(1) $p_1 \supseteq p_2 \Leftrightarrow \hat{\mu}_{V1} \geq \hat{\mu}_{V2},\ \hat{\mu}_{C1} \geq \hat{\mu}_{C2},\ \hat{\nu}_{V1} \leq \hat{\nu}_{V2},\ \hat{\nu}_{C1} \leq \hat{\nu}_{C2}$
(2) $p_1 = p_2 \Leftrightarrow p_1 \supseteq p_2\ and\ p_2 \supseteq p_1$
(3) $p_1 \cup p_2 = \langle(\hat{\mu}_{V1} \vee \hat{\mu}_{V2}, \hat{\mu}_{C1} \vee \hat{\mu}_{C2}), (\hat{\nu}_{V1} \wedge \hat{\nu}_{V2}, \hat{\nu}_{C1} \wedge \hat{\nu}_{C2})\rangle$
(4) $p_1 \cap p_2 = \langle(\hat{\mu}_{V1} \wedge \hat{\mu}_{V2}, \hat{\mu}_{C1} \wedge \hat{\mu}_{C2}), (\hat{\nu}_{V1} \vee \hat{\nu}_{V2}, \hat{\nu}_{C1} \vee \hat{\nu}_{C2})\rangle$
(5) $(p_1)^c = \langle(\hat{\nu}_{V1}, \hat{\nu}_{C1}), (\hat{\mu}_{V1}, \hat{\mu}_{C1})\rangle$
(6) $p_1 \oplus p_2 = \langle(\hat{\mu}_{V1} + \hat{\mu}_{V2} - \hat{\mu}_{V1}\hat{\mu}_{V2}, \hat{\mu}_{C1} + \hat{\mu}_{C2} - \hat{\mu}_{C1}\hat{\mu}_{C2}), (\hat{\nu}_{V1}\hat{\nu}_{V2}, \hat{\nu}_{C1}\hat{\nu}_{C2})\rangle$
(7) $p_1 \otimes p_2 = \langle(\hat{\mu}_{V1}\hat{\mu}_{V2}, \hat{\mu}_{C2}\hat{\mu}_{C2}), (\hat{\nu}_{V1} + \hat{\nu}_{V2} - \hat{\nu}_{V1}\hat{\nu}_{V2}, \hat{\nu}_{C1} + \hat{\nu}_{C2} - \hat{\nu}_{C1}\hat{\nu}_{C2})\rangle$
(8) $\eta p_1 = \langle(1 - (1 - \hat{\mu}_{V1})^\eta, 1 - (1 - \hat{\mu}_{C1})^\eta), (\hat{\nu}_{V1}^\eta, \hat{\nu}_{R1}^\eta)\rangle$
(9) $p_1^\eta = \langle(\hat{\mu}_{V1}^\eta, \hat{\mu}_{R1}^\eta), (1 - (1 - \hat{\nu}_{V1})^\eta, 1 - (1 - \hat{\nu}_{C1})^\eta)\rangle$

A score function is a tool that is used in decision-making to transform fuzzy values into a single numeric score that reflects the straightforward ranking of alternatives. The score function for comparing IFCNs $p_j = \langle(\hat{\mu}_{Vj}, \hat{\mu}_{Cj}), (\hat{\nu}_{Vj}, \hat{\nu}_{Cj})\rangle\ j = 1, 2$, is given below.

$$\mathscr{Y}(p_j) = \hat{\mu}_{Vj}\hat{\mu}_{Cj} - \hat{\nu}_V j\hat{\nu}_{Cj} \tag{2. 1}$$

Fuzzy values are often evaluated by an accuracy function in the context of a scoring function with respect to accuracy as well as in decision-making. Below gives the accuracy function for two IFCNs.

$$\mathscr{H}(p_j) = \hat{\mu}_{Vj}\hat{\mu}_{Cj} + \hat{\nu}_{Vj}\hat{\nu}_{Cj} \tag{2. 2}$$

The following results can be followed by applying the score function.

- If $\mathscr{Y}(p_1) < \mathscr{Y}(p_2)$, then $p_1 < p_2$.
- If $\mathscr{Y}(p_1) = \mathscr{Y}(p_2)$, then
  (1) If $\mathscr{H}(p_1) = \mathscr{H}(p_2)$, then $\hat{\mu}_{V1}\hat{\mu}_{C1} = \hat{\mu}_{V2}\hat{\mu}_{C2}$ and $\hat{\nu}_{V1}\hat{\nu}_{C1} = \hat{\nu}_{V2}\hat{\nu}_{C2}$.
  (2) If $\mathscr{H}(p_1) < \mathscr{H}(p_2)$, then $p_1 < p_2$.
  (3) If $\mathscr{H}(p_1) > \mathscr{H}(p_2)$, then $p_1 > p_2$.

3. GIFCNWA OPERATOR

The GIFCNWA operator is a robust mathematical tool that aims to aggregate information in a decision-making environment that is uncertain as well as vague. The operator, built on the IFCNs, effectively incorporates membership, non-membership, hesitation degrees, and credibility measures into the comprehensive aggregation. The GIFCNWA operator utilises varying attribute weights and dependencies, thereby providing flexibility to the underlying decision-makers to allocate resources in a balanced manner. In fact, it excels in this regard by its capacity to simultaneously tackle its fuzziness of data and the reliability of evaluations which make it particularly suited for MADM problems. By delivering an effective mechanism for extracting meaningful information, the GIFCNWA operator can help achieve more accurate and reliable decisions in uncertain situations.

Definition 3.1. Let $GIFCNWA : \eta^{\hbar} \rightarrow \eta$, if

$$GIFCNWA(\alpha_1, \alpha_2, ..., \alpha_{\hbar}) = (\xi_1 \alpha_1^{\eta} \oplus \xi_2 \alpha_2^{\eta} \oplus ... \oplus \xi_{\hbar} \alpha_{\hbar}^{\eta})^{\frac{1}{\eta}}, \tag{3. 3}$$

then the function GIFCNWA is called generalized intuitionistic fuzzy credibility numbers weighted averaging operator, where $\eta > 0$, $\xi = (\xi_1, \xi_2, ..., \xi_{\hbar})^T$ being a weighted vector of $\alpha_j (j = 1, 2, ..., \hbar)$, with $\xi_j \in [0, 1], j = 1, 2, ..., \hbar$ and $\sum_{j=1}^{\hbar} \xi_j = 1$.

Now, we prove some mathematical properties of this operator. First, we provide the aggregated value of the proposed operator in the following theorem.

Theorem 3.2. The aggregated value by using GIFCNWA operator is an intuitionistic fuzzy credibility number, and

$$GIFCNWA(\alpha_1, \alpha_2, ..., \alpha_{\hbar}) = \Bigg\langle \Bigg(\big((1 - \prod_{j=1}^{\hbar} (1 - \hat{\mu}_{V_{\alpha j}}^{\eta})^{\xi_j}\big)^{\frac{1}{\eta}}, \big(1 - \prod_{j=1}^{\hbar} (1 - \hat{\mu}_{C_{\alpha j}}^{\eta})^{\xi_j}\big)^{\frac{1}{\eta}} \Bigg),$$
$$\Bigg(1 - \big(1 - \prod_{j=1}^{\hbar} (1 - (1 - \hat{\nu}_{V_{\alpha j}})^{\eta})^{\xi_j}\big)^{\frac{1}{\eta}}, 1 - \big(1 - \prod_{j=1}^{\hbar} (1 - (1 - \hat{\nu}_{V_{\alpha j}})^{\eta})^{\xi_j}\big)^{\frac{1}{\eta}} \Bigg) \Bigg\rangle \tag{3. 4}$$

Proof. The first result follows directly from Definition 2.2. In the subsequent section, we will first establish the proof

$$\xi_1 \alpha_1^{\eta} \oplus \xi_2 \alpha_2^{\eta} \oplus ... \oplus \xi_{\hbar} \alpha_{\hbar}^{\eta} = \Bigg\langle \Bigg(1 - \prod_{j=1}^{\hbar} (1 - \hat{\mu}_{V_{\alpha j}}^{\eta})^{\xi_j}, 1 - \prod_{j=1}^{\hbar} (1 - \hat{\mu}_{C_{\alpha j}}^{\eta})^{\xi_j} \Bigg),$$
$$\Bigg(\prod_{j=1}^{\hbar} (1 - (1 - \hat{\nu}_{V_{\alpha j}})^{\eta})^{\xi_j}, \prod_{j=1}^{\hbar} (1 - (1 - \hat{\nu}_{C_{\alpha j}})^{\eta})^{\xi_j} \Bigg) \Bigg\rangle \tag{3. 5}$$

by using mathematical induction on $\hbar$:
For $\hbar = 2$: Since
$\alpha_1^{\eta} = \Bigg\langle \Bigg(\hat{\mu}_{V_{\alpha 1}}^{\eta}, \hat{\mu}_{C_{\alpha 1}}^{\eta} \Bigg), \Bigg(1 - (1 - \hat{\nu}_{V_{\alpha 1}})^{\eta}, (1 - (1 - \hat{\nu}_{C_{\alpha 1}})^{\eta} \Bigg) \Bigg\rangle$ and

$\alpha_2^\eta = \left\langle \left(\hat{\mu}^\eta_{V_{\alpha 2}}, \hat{\mu}^\eta_{C_{\alpha 2}}\right), \left(1-(1-\hat{\nu}_{V_{\alpha 2}})^\eta, (1-(1-\hat{\nu}_{C_{\alpha 2}})^\eta\right)\right\rangle$
then

$$\xi_1\alpha_1^\eta \oplus \xi_2\alpha_2^\eta = \Bigg\langle \left(1-\prod_{j=1}^{2}(1-\hat{\mu}^\eta_{V_{\alpha j}})^{\xi_j}, (1-\prod_{j=1}^{2}(1-\hat{\mu}^\eta_{C_{\alpha j}})^{\xi_j}\right), \left(\prod_{j=1}^{2}(1-(1-\hat{\nu}_{V_{\alpha j}})^\eta)^{\xi_j}, \prod_{j=1}^{2}(1-(1-\hat{\nu}_{C_{\alpha j}})^\eta)^{\xi_j}\right)\Bigg\rangle$$

If 3. 5 holds for $\hbar = k$, that is

$$\xi_1\alpha_1^\eta \oplus \xi_2\alpha_2^\eta \oplus ... \oplus \xi_k\alpha_k^\eta = \Bigg\langle \left(1-\prod_{j=1}^{k}(1-\hat{\mu}^\eta_{V_{\alpha j}})^{\xi_j}, 1-\prod_{j=1}^{k}(1-\hat{\mu}^\eta_{C_{\alpha j}})^{\xi_j}\right), \left(\prod_{j=1}^{k}(1-(1-\hat{\nu}_{V_{\alpha j}})^\eta)^{\xi_j}, \prod_{j=1}^{k}(1-(1-\hat{\nu}_{C_{\alpha j}})^\eta)^{\xi_j}\right)\Bigg\rangle$$

Then, when $\hbar = k+1$, according to the operational rules 6, 8, and 9 in Definition 2.2, we obtain

$$\xi_1\alpha_1^\eta \oplus \xi_2\alpha_2^\eta \oplus ... \oplus \xi_{k+1}\alpha_{k+1}^\eta = \Bigg\langle \left(1-\prod_{j=1}^{k}(1-\hat{\mu}^\eta_{V_{\alpha j}})^{\xi_j}, 1-\prod_{j=1}^{k}(1-\hat{\mu}^\eta_{C_{\alpha j}})^{\xi_j}\right), \left(\prod_{j=1}^{k}(1-(1-\hat{\nu}_{V_{\alpha j}})^\eta)^{\xi_j}, \prod_{j=1}^{k}(1-(1-\hat{\nu}_{C_{\alpha j}})^\eta)^{\xi_j}\right)\Bigg\rangle \oplus \Bigg\langle \left(1-(1-\hat{\mu}^\eta_{V_{\alpha j}})^{\xi_j}, 1-(1-\hat{\mu}^\eta_{C_{\alpha j}})^{\xi_j}\right), \left((1-(1-\hat{\nu}_{V_{\alpha j}})^\eta)^{\xi_j}, (1-(1-\hat{\nu}_{C_{\alpha j}})^\eta)^{\xi_j}\right)\Bigg\rangle$$

$$\xi_1\alpha_1^\eta \oplus \xi_2\alpha_2^\eta \oplus ... \oplus \xi_{k+1}\alpha_{k+1}^\eta = \Bigg\langle \left(1-\prod_{j=1}^{k+1}(1-\hat{\mu}^\eta_{V_{\alpha j}})^{\xi_j}, 1-\prod_{j=1}^{k+1}(1-\hat{\mu}^\eta_{C_{\alpha j}})^{\xi_j}\right), \left(\prod_{j=1}^{k+1}(1-(1-\hat{\nu}_{V_{\alpha j}})^\eta)^{\xi_j}, \prod_{j=1}^{k+1}(1-(1-\hat{\nu}_{C_{\alpha j}})^\eta)^{\xi_j}\right)\Bigg\rangle$$

i.e, Eq. 3. 5 holds for $n = k+1$. Thus, Eq. 3. 5 holds for all $\hbar$. Then

$$GIFCNWA(\alpha_1, \alpha_2, ..., \alpha_\hbar) = \Bigg\langle \Bigg[\left((1-\prod_{j=1}^{\hbar}(1-\hat{\mu}^\eta_{V_{\alpha j}})^{\xi_j}, 1-\prod_{j=1}^{\hbar}(1-\hat{\mu}^\eta_{C_{\alpha j}})^{\xi_j}\right), \left(\prod_{j=1}^{\hbar}(1-(1-\hat{\nu}_{V_{\alpha j}})^\eta)^{\xi_j}, \prod_{j=1}^{\hbar}(1-(1-\hat{\nu}_{C_{\alpha j}})^\eta)^{\xi_j}\right)\Bigg]^{\frac{1}{\eta}} \Bigg\rangle$$

$$GIFCNWA(\alpha_1,\alpha_2,...,\alpha_\hbar)=\left\langle\left((1-\prod_{j=1}^{\hbar}(1-\hat{\mu}^{\eta}_{V_{\alpha j}})^{\xi_j},1-\prod_{j=1}^{\hbar}(1-\hat{\mu}^{\eta}_{C_{\alpha j}})^{\xi_j}\right)^{\frac{1}{\eta}},\right.$$
$$\left.\left(1-\big(1-\prod_{j=1}^{\hbar}(1-(1-\hat{\nu}_{V_{\alpha j}})^{\eta}\big)^{\xi_j},1-\big(1-\prod_{j=1}^{\hbar}(1-(1-\hat{\nu}_{C_{\alpha_j}})^{\eta}\big)^{\xi_j}\right)^{\frac{1}{\eta}}\right\rangle$$

□

Hence, the aggregated result obtained through the GIFCNWA operator confirms that the operator preserves the fundamental structure and properties of IFCNs.

Theorem 3.3. If all IFCNs $\alpha_j(j=1,2,...,\hbar)$ are equal, i.e. $\alpha_j=\alpha$, for all j, then

$$GIFCNWA(\alpha_1,\alpha_2,....\alpha_\hbar)=\alpha$$

Proof.

$$\begin{aligned}GIFCNWA(\alpha_1,\alpha_2,....\alpha_\hbar)&=(\xi_1\alpha_1^{\eta}\oplus\xi_2\alpha_2^{\eta}\oplus...\oplus\xi_\hbar\alpha_\hbar^{\eta})^{\frac{1}{\eta}}\\&=(\xi_1\alpha^{\eta}\oplus\xi\alpha^{\eta}\oplus...\oplus\xi\alpha^{\eta})^{\frac{1}{\eta}}\\&=((\xi_1+\xi_2+...+\xi_\hbar)\alpha^{\eta})^{\frac{1}{\eta}}\\&=(\alpha^{\eta})^{\frac{1}{\eta}}=\alpha\end{aligned}$$

□

This theorem demonstrates that the operator GIFCNWA exhibits idempotency, where the same input consistently yields the same aggregated output.

Theorem 3.4. Let

$\alpha^-=\left\langle\left(min_j(\hat{\mu}_{V_{\alpha j}},\hat{\mu}_{C_{\alpha j}})\right),\left(max_j(\hat{\nu}_{V_{\alpha j}},\hat{\nu}_{C_{\alpha j}})\right)\right\rangle,$

$\alpha^+=\left\langle\left(max_j(\hat{\mu}_{V_{\alpha j}},\hat{\mu}_{C_{\alpha j}})\right),\left(min_j(\hat{\nu}_{V_{\alpha j}},\hat{\nu}_{C_{\alpha j}})\right)\right\rangle$

Then

$$\alpha^-\le GIFCNWA(\alpha_1,\alpha_2,....\alpha_\hbar)\le\alpha^+ \tag{3. 6}$$

Proof. Since $min_j(\hat{\mu}_{V_{\alpha j}})\le\hat{\mu}_{V_{\alpha j}}\le max_j(\hat{\mu}_{V_{\alpha j}})$ and $min_j(\hat{\mu}_{C_{\alpha j}})\le\hat{\mu}_{C_{\alpha j}}\le max_j(\hat{\mu}_{C_{\alpha j}})$, $min_j(\hat{\nu}_{V_{\alpha j}})\le\hat{\nu}_{V_{\alpha j}}\le max_j(\hat{\nu}_{V_{\alpha j}})$ and $min_j(\hat{\nu}_{C_{\alpha j}})\le\hat{\nu}_{C_{\alpha j}}\le max_j(\hat{\nu}_{C_{\alpha j}})$, for all j, then

$$\prod_{j=1}^{\hbar}(1-\hat{\mu}^{\eta}_{V_{\alpha j}})^{w_j}\ge\prod_{j=1}^{\hbar}\left(1-\left(max_j(\hat{\mu}_{V_{\alpha j}})\right)^{\eta}\right)^{\xi_j}=1-\left(max_j(\hat{\mu}_{V_{\alpha j}})\right)^{\eta}$$

and then

$$\left(1-\prod_{j=1}^{\hbar}(1-\hat{\mu}^{\eta}_{V_{\alpha j}})^{w_j}\right)^{\frac{1}{\eta}}\le max_j\hat{\mu}_{V_{\alpha j}} \tag{3. 7}$$

Similarly, we have

$$\left(1-\prod_{j=1}^{\hbar}(1-\hat{\mu}^{\eta}_{V_{\alpha j}})^{w_j}\right)^{\frac{1}{\eta}}\ge min_j\hat{\mu}_{V_{\alpha j}} \tag{3. 8}$$

In the same way, we can get

$$\left(1-\prod_{j=1}^{\hbar}(1-\hat{\mu}_{C_{\alpha j}}^{\eta})^{w_j}\right)^{\frac{1}{\eta}} \leq max_j\hat{\mu}_{C_{\alpha j}} \tag{3. 9}$$

And

$$\left(1-\prod_{j=1}^{\hbar}(1-\hat{\mu}_{C_{\alpha j}}^{\eta})^{w_j}\right)^{\frac{1}{\eta}} \geq min_j\hat{\mu}_{C_{\alpha j}} \tag{3. 10}$$

Now,

$$\prod_{j=1}^{\hbar}\left(1-\left(1-\hat{\nu}_{V_{\alpha j}}\right)^{\eta}\right)^{\xi_j} \leq \prod_{j=1}^{\hbar}\left(1-(1-max_j(\hat{\nu}_{V_{\alpha j}}))^{\eta}\right)^{\xi_j} = 1-\left(1-max_j(\hat{\nu}_{V_{\alpha j}})\right)^{\eta}$$

$$1-\prod_{j=1}^{\hbar}\left(1-(1-\hat{\nu}_{V_{\alpha j}})^{\eta}\right)^{\xi_j} \geq \left((1-max_j(\hat{\nu}_{V_{\alpha j}})\right)^{\eta}$$

$$\left[\left(1-\prod_{j=1}^{\hbar}\left(1-(1-\hat{\nu}_{V_{\alpha j}})^{\eta}\right)^{\xi_j}\right)\right]^{\frac{1}{\eta}} \geq 1-max_j(\hat{\nu}_{V_{\alpha j}})$$

$$1-\left[\left(1-\prod_{j=1}^{\hbar}\left(1-(1-\hat{\nu}_{V_{\alpha j}})^{\eta}\right)^{\xi_j}\right)\right]^{\frac{1}{\eta}} \leq max_j(\hat{\nu}_{V_{\alpha j}}) \tag{3. 11}$$

Similarly,

$$1-\left[\left(1-\prod_{j=1}^{\hbar}\left(1-(1-\hat{\nu}_{V_{\alpha j}})^{\eta}\right)^{\xi_j}\right)\right]^{\frac{1}{\eta}} \geq min_j(\hat{\nu}_{V_{\alpha j}}) \tag{3. 12}$$

In the same way, we can get

$$1-\left[\left(1-\prod_{j=1}^{\hbar}\left(1-(1-\hat{\nu}_{C_{\alpha j}})^{\eta}\right)^{\xi_j}\right)\right]^{\frac{1}{\eta}} \leq max_j(\hat{\nu}_{C_{\alpha j}}) \tag{3. 13}$$

And

$$1-\left[\left(1-\prod_{j=1}^{\hbar}\left(1-(1-\hat{\nu}_{C_{\alpha j}})^{\eta}\right)^{\xi_j}\right)\right]^{\frac{1}{\eta}} \geq min_j(\hat{\nu}_{C_{\alpha j}}) \tag{3. 14}$$

Let $GIFCNWA(\alpha_1,\alpha_2,....\alpha_\hbar)=\alpha=\left\langle(\hat{\mu}_{V_{\alpha j}},\hat{\mu}_{C_{\alpha j}}),(\hat{\nu}_{V_{\alpha j}},\hat{\nu}_{C_{\alpha j}})\right\rangle$, then

$$(\hat{\mu}_{V_{\alpha j}},\hat{\mu}_{C_{\alpha j}}) \leq max(\hat{\mu}_{V_{\alpha_j}},\hat{\mu}_{C_{\alpha_j}}) \tag{3. 15}$$

$$(\hat{\nu}_{V_{\alpha j}},\hat{\nu}_{C_{\alpha j}}) \geq min(\hat{\nu}_{V_{\alpha j}},\hat{\nu}_{C_{\alpha j}}) \tag{3. 16}$$

From inequalities 3. 15 and 3. 16 , we have

$$\mathscr{Y}(\alpha)=\hat{\mu}_{V_{\alpha j}}\hat{\mu}_{C_{\alpha j}}-\hat{\nu}_{V_{\alpha j}}\hat{\nu}_{C_{\alpha j}} \leq max(\hat{\mu}_{V_{\alpha j}}\hat{\mu}_{C_{\alpha j}})-min(\hat{\nu}_{V_{\alpha j}}\hat{\nu}_{C_{\alpha j}})=\mathscr{Y}(\alpha^+)$$

$$\mathscr{Y}(\alpha)=\hat{\mu}_{V_{\alpha j}}\hat{\mu}_{C_{\alpha j}}-\hat{\nu}_{V_{\alpha j}}\hat{\nu}_{C_{\alpha j}} \geq min(\hat{\mu}_{V_{\alpha j}}\hat{\mu}_{C_{\alpha j}})-max(\hat{\nu}_{V_{\alpha j}}\hat{\nu}_{C_{\alpha j}})=\mathscr{Y}(\alpha^-)$$

If $\mathscr{Y}(\alpha) < \mathscr{Y}(\alpha^{+})$ and $\mathscr{Y}(\alpha) < \mathscr{Y}(\alpha^{-})$, then we get by the score function 2. 1 , that is

$$\alpha^{-} \leq GIFCNWA(\alpha_1, \alpha_2, ...\alpha_\hbar) \leq \alpha^{+} \tag{3. 17}$$

If $\mathscr{Y}(\alpha) = \mathscr{Y}(\alpha^{+})$, i.e. $\hat{\mu}_{V_{\alpha j}}\hat{\mu}_{C_{\alpha j}} - \hat{\nu}_{V_{\alpha j}}\hat{\nu}_{C_{\alpha j}} = max(\hat{\mu}_{V_{\alpha j}}\hat{\mu}_{C_{\alpha j}}) - min(\hat{\nu}_{V_{\alpha j}}\hat{\nu}_{C_{\alpha j}})$, then by Equations 3. 7 , 3. 9 , 3. 12 and 3. 14 , we have

$$\hat{\mu}_{V_{\alpha j}}\hat{\mu}_{C_{\alpha j}} = max(\hat{\mu}_{V_{\alpha j}}\hat{\mu}_{C_{\alpha j}}), \quad \hat{\nu}_{V_{\alpha j}}\hat{\nu}_{C_{\alpha j}} = min(\hat{\nu}_{V_{\alpha j}}\hat{\nu}_{C_{\alpha j}})$$

So,

$$\mathscr{H}(\alpha) = \hat{\mu}_{V_{\alpha j}}\hat{\mu}_{C_{\alpha j}} + \hat{\nu}_{V_{\alpha j}}\hat{\nu}_{C_{\alpha j}} = max(\hat{\mu}_{V_{\alpha j}}\hat{\mu}_{C_{\alpha j}}) + min(\hat{\nu}_{V_{\alpha j}}\hat{\nu}_{C_{\alpha j}}) = \mathscr{H}(\alpha^{+})$$

Then it follows from the Definition 2.2, that is

$$GIFCNWA(\alpha_1, \alpha_2, ..., \alpha_\hbar) = \alpha^{+} \tag{3. 18}$$

If $\mathscr{Y}(\alpha) = \mathscr{Y}(\alpha^{-})$, i.e. $\hat{\mu}_{V_{\alpha j}}\hat{\mu}_{C_{\alpha j}} - \hat{\nu}_{V_{\alpha j}}\hat{\nu}_{C_{\alpha j}} = min(\hat{\mu}_{V_{\alpha j}}\hat{\mu}_{C_{\alpha j}}) - max(\hat{\nu}_{V_{\alpha j}}\hat{\nu}_{C_{\alpha j}})$, then by Equations 3. 8 , 3. 10 , 3. 12 and 3. 14 we obtain

$$\hat{\mu}_{V_{\alpha j}}\hat{\mu}_{C_{\alpha j}} = min(\hat{\mu}_{V_{\alpha j}}\hat{\mu}_{C_{\alpha j}}), \quad \hat{\nu}_{V_{\alpha j}}\hat{\nu}_{C_{\alpha j}} = max(\hat{\nu}_{V_{\alpha j}}\hat{\nu}_{C_{\alpha j}})$$

So

$$\mathscr{H}(\alpha) = \hat{\mu}_{V_{\alpha j}}\hat{\mu}_{C_{\alpha j}} + \hat{\nu}_{V_{\alpha j}}\hat{\nu}_{C_{\alpha j}} = min(\hat{\mu}_{V_{\alpha j}}\hat{\mu}_{C_{\alpha j}}) + max(\hat{\nu}_{V_{\alpha j}}\hat{\nu}_{C_{\alpha j}}) = \mathscr{H}(\alpha^{-})$$

Thus, it follows from Definition 2.2, that is

$$GIFCNWA(\alpha_1, \alpha_2, ..., \alpha_\hbar) = \alpha^{-} \tag{3. 19}$$

Therefore, from Equations 3. 17 - 3. 19 , we know that Equation 3. 6 holds always. □

This theorem ensures that the output of the GIFCNWA operator is constrained by defined limits based on the input IFCNs.

Theorem 3.5. Let $\alpha_j = \left\langle (\hat{\mu}_{V_{\alpha j}}, \hat{\mu}_{C_{\alpha j}}), (\hat{\nu}_{V_{\alpha j}}, \hat{\nu}_{C_{\alpha j}}) \right\rangle (j = 1, 2, ..., \hbar)$ and $\alpha^*_j = \left\langle (\hat{\mu}_{V_{\alpha^* j}}, \hat{\mu}_{C_{\alpha^* j}}), (\hat{\nu}_{V_{\alpha^* j}}, \hat{\nu}_{C_{\alpha^* j}}) \right\rangle (j = 1, 2, ..., \hbar)$ be two collections of IFCNs, $\eta > 0$. If $\hat{\mu}_{V_{\alpha j}} \leq \hat{\mu}_{V_{\alpha^* j}}, \hat{\mu}_{C_{\alpha j}} \leq \hat{\mu}_{C_{\alpha^* j}}, \hat{\nu}_{V_{\alpha j}} \geq \hat{\nu}_{V_{\alpha^* j}}, \hat{\nu}_{C_{\alpha j}} \geq \hat{\nu}_{C_{\alpha^* j}}$, for all j, then

$$GIFCNWA(\alpha_1, \alpha_2, ...\alpha_\hbar) \leq GIFCNWA(\alpha^*_1, \alpha^*_2, ...\alpha^*_\hbar) \tag{3. 20}$$

Proof. Since $\hat{\mu}_{V_{\alpha j}} \leq \hat{\mu}_{V_{\alpha^* j}}$, for all j, then

$$\prod_{j=1}^{\hbar} \left(1 - \hat{\mu}^{\eta}_{V_{\alpha j}}\right)^{\xi_j} \geq \prod_{j=1}^{\hbar} \left(1 - \hat{\mu}^{\eta}_{V_{\alpha^* j}}\right)^{\xi_j}$$

$$1 - \prod_{j=1}^{\hbar} \left(1 - \hat{\mu}^{\eta}_{V_{\alpha j}}\right)^{\xi_j} \leq 1 - \prod_{j=1}^{\hbar} \left(1 - \hat{\mu}^{\eta}_{V_{\alpha^* j}}\right)^{\xi_j}$$

$$\left(1 - \prod_{j=1}^{\hbar} \left(1 - \hat{\mu}^{\eta}_{V_{\alpha j}}\right)^{\xi_j}\right)^{\frac{1}{\eta}} \leq \left(1 - \prod_{j=1}^{\hbar} \left(1 - \hat{\mu}^{\eta}_{V_{\alpha^* j}}\right)^{\xi_j}\right)^{\frac{1}{\eta}} \tag{3. 21}$$

Similarly, for all j, $\hat{\mu}_{C_{\alpha j}} \leq \hat{\mu}_{C_{\alpha^* j}}$, we have

$$\left(1 - \prod_{j=1}^{\hbar} \left(1 - \hat{\mu}^{\eta}_{C_{\alpha j}}\right)^{\xi_j}\right)^{\frac{1}{\eta}} \leq \left(1 - \prod_{j=1}^{\hbar} \left(1 - \hat{\mu}^{\eta}_{C_{\alpha^* j}}\right)^{\xi_j}\right)^{\frac{1}{\eta}} \tag{3. 22}$$

Now, since $\hat{\nu}_{V_{\alpha j}} \geq \hat{\nu}_{V_{\alpha^* j}}$, for all j, then

$$\prod_{j=1}^{\hbar} \left(1-(1-\hat{\nu}_{V_{\alpha j}})^{\eta}\right)^{\xi_j} \geq \prod_{j=1}^{\hbar} \left(1-(1-\hat{\nu}_{V_{\alpha^* j}})^{\eta}\right)^{\xi_j}$$

$$1-\prod_{j=1}^{\hbar} \left(1-(1-\hat{\nu}_{V_{\alpha j}})^{\eta}\right)^{\xi_j} \leq 1-\prod_{j=1}^{\hbar} \left(1-(1-\hat{\nu}_{V_{\alpha^* j}})^{\eta}\right)^{\xi_j}$$

$$\left(1-\prod_{j=1}^{\hbar} \left(1-(1-\hat{\nu}_{V_{\alpha j}})^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}} \leq \left(1-\prod_{j=1}^{\hbar} \left(1-(1-\hat{\nu}_{V_{\alpha^* j}})^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}}$$

$$1-\left(1-\prod_{j=1}^{\hbar} \left(1-(1-\hat{\nu}_{V_{\alpha j}})^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}} \geq 1-\left(1-\prod_{j=1}^{\hbar} \left(1-(1-\hat{\nu}_{V_{\alpha^* j}})^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}} \quad (3.\ 23)$$

Similarly, for all j, $\hat{\nu}_{C_{\alpha j}} \geq \hat{\nu}_{C_{\alpha^* j}}$, we have

$$1-\left(1-\prod_{j=1}^{\hbar} \left(1-(1-\hat{\nu}_{C_{\alpha j}})^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}} \geq 1-\left(1-\prod_{j=1}^{\hbar} \left(1-(1-\hat{\nu}_{C_{\alpha^* j}})^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}} \quad (3.\ 24)$$

According to the inequality 3. 21 and 3. 22 ,

$$\left(\left(1-\prod_{j=1}^{\hbar} \left(1-\hat{\mu}_{V_{\alpha j}}^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}}, \left(1-\prod_{j=1}^{\hbar} \left(1-\hat{\mu}_{C_{\alpha j}}^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}}\right) \geq \left(\left(1-\prod_{j=1}^{\hbar} \left(1-\hat{\mu}_{V_{\alpha^* j}}^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}}, \left(1-\prod_{j=1}^{\hbar} \left(1-\hat{\mu}_{C_{\alpha^* j}}^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}}\right) \quad (3.\ 25)$$

According to the inequality 3. 23 and 3. 24 ,

$$\left(1-\left(1-\prod_{j=1}^{\hbar} \left(1-(1-\hat{\nu}_{V_{\alpha j}})^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}}, 1-\left(1-\prod_{j=1}^{\hbar} \left(1-(1-\hat{\nu}_{C_{\alpha j}})^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}}\right) \geq \left(1-\left(1-\prod_{j=1}^{\hbar} \left(1-(1-\hat{\nu}_{V_{\alpha^* j}})^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}}, 1-\left(1-\prod_{j=1}^{\hbar} \left(1-(1-\hat{\nu}_{C_{\alpha^* j}})^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}}\right) \quad (3.\ 26)$$

Thus, by using the inequalities 3. 25 and 3. 26 we have

$$\Big(\big(1-\prod_{j=1}^{\hbar}(1-\hat{\mu}^{\eta}_{V_{\alpha j}})^{\xi_j}\big)^{\frac{1}{\eta}},\big(1-\prod_{j=1}^{\hbar}(1-\hat{\mu}^{\eta}_{C_{\alpha j}})^{\xi_j}\big)^{\frac{1}{\eta}}\Big)-$$
$$\Big(1-\big(1-\prod_{j=1}^{\hbar}\big(1-(1-\hat{\nu}_{V_{\alpha j}})^{\eta}\big)^{\xi_j}\big)^{\frac{1}{\eta}},1-\big(1-\prod_{j=1}^{\hbar}\big(1-(1-\hat{\nu}_{C_{\alpha j}})^{\eta}\big)^{\xi_j}\big)^{\frac{1}{\eta}}\Big)\leq$$
$$\Big(\big(1-\prod_{j=1}^{\hbar}\big(1-\hat{\mu}^{\eta}_{V_{\alpha^* j}}\big)^{\xi_j}\big)^{\frac{1}{\eta}},\big(1-\prod_{j=1}^{\hbar}\big(1-\hat{\mu}^{\eta}_{C_{\alpha^* j}}\big)^{\xi_j}\big)^{\frac{1}{\eta}}\Big)-$$
$$\Big(1-\big(1-\prod_{j=1}^{\hbar}\big(1-(1-\hat{\nu}_{V_{\alpha^* j}})^{\eta}\big)^{\xi_j}\big)^{\frac{1}{\eta}},1-\big(1-\prod_{j=1}^{\hbar}\big(1-(1-\hat{\nu}_{C_{\alpha^* j}})^{\eta}\big)^{\xi_j}\big)^{\frac{1}{\eta}}\Big) \tag{3. 27}$$

Let $\alpha = GIFCNWA(\alpha_1, \alpha_2, ..., \alpha_\hbar)$ and $\alpha^* = GIFCNWA(\alpha_1^*, \alpha_2^*, ..., \alpha_\hbar^*)$.
Then using the inequality 3. 27 , we have

$$\Big(\big(1-\prod_{j=1}^{\hbar}(1-\hat{\mu}^{\eta}_{V_{\alpha j}})^{\xi_j}\big)^{\frac{1}{\eta}}\Big).\Big(\big(1-\prod_{j=1}^{\hbar}(1-\hat{\mu}^{\eta}_{C_{\alpha j}})^{\xi_j}\big)^{\frac{1}{\eta}}\Big)-$$
$$\Big(1-\big(1-\prod_{j=1}^{\hbar}\big(1-(1-\hat{\nu}_{V_{\alpha j}})^{\eta}\big)^{\xi_j}\big)^{\frac{1}{\eta}}\Big).\Big(1-\big(1-\prod_{j=1}^{\hbar}\big(1-(1-\hat{\nu}_{C_{\alpha j}})^{\eta}\big)^{\xi_j}\big)^{\frac{1}{\eta}}\Big)\leq$$
$$\Big(\big(1-\prod_{j=1}^{\hbar}\big(1-\hat{\mu}^{\eta}_{V_{\alpha^* j}}\big)^{\xi_j}\big)^{\frac{1}{\eta}}\Big).\Big(\big(1-\prod_{j=1}^{\hbar}\big(1-\hat{\mu}^{\eta}_{C_{\alpha^* j}}\big)^{\xi_j}\big)^{\frac{1}{\eta}}\Big)-$$
$$\Big(1-\big(1-\prod_{j=1}^{\hbar}\big(1-(1-\hat{\nu}_{V_{\alpha^* j}})^{\eta}\big)^{\xi_j}\big)^{\frac{1}{\eta}}\Big).\Big(1-\big(1-\prod_{j=1}^{\hbar}\big(1-(1-\hat{\nu}_{C_{\alpha^* j}})^{\eta}\big)^{\xi_j}\big)^{\frac{1}{\eta}}\Big) \tag{3. 28}$$

Thus, from Equation 3. 28 , We have $\mathscr{Y}(\alpha) \leq \mathscr{Y}(\alpha^*)$.
If $\mathscr{Y}(\alpha) < \mathscr{Y}(\alpha^*)$, then by Definition 2.2, we can get

$$GIFCNWA(\alpha_1, \alpha_2, ..., \alpha_\hbar) < GIFCNWA(\alpha_1^*, \alpha_2^*, ..., \alpha_\hbar^*) \tag{3. 29}$$

If $\mathscr{Y}(\alpha) = \mathscr{Y}(\alpha^*)$, then

$$\left(\left(1-\prod_{j=1}^{\hbar}(1-\hat{\mu}^{\eta}_{V_{\alpha j}})^{\xi_j}\right)^{\frac{1}{\eta}}\right).\left(\left(1-\prod_{j=1}^{\hbar}(1-\hat{\mu}^{\eta}_{C_{\alpha j}})^{\xi_j}\right)^{\frac{1}{\eta}}\right)-$$

$$\left(1-\left(1-\prod_{j=1}^{\hbar}\left(1-(1-\hat{\nu}_{V_{\alpha j}})^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}}\right).\left(1-\left(1-\prod_{j=1}^{\hbar}\left(1-(1-\hat{\nu}_{C_{\alpha j}})^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}}\right)=$$

$$\left(\left(1-\prod_{j=1}^{\hbar}\left(1-\hat{\mu}^{\eta}_{V_{\alpha^* j}}\right)^{\xi_j}\right)^{\frac{1}{\eta}}\right).\left(\left(1-\prod_{j=1}^{\hbar}\left(1-\hat{\mu}^{\eta}_{C_{\alpha^* j}}\right)^{\xi_j}\right)^{\frac{1}{\eta}}\right)-$$

$$\left(1-\left(1-\prod_{j=1}^{\hbar}\left(1-(1-\hat{\nu}_{V_{\alpha^* j}})^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}}\right).\left(1-\left(1-\prod_{j=1}^{\hbar}\left(1-(1-\hat{\nu}_{C_{\alpha^* j}})^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}}\right)$$

Since $\hat{\mu}_{V_{\alpha j}}\hat{\mu}_{C_{\alpha j}} \leq \hat{\mu}_{V_{\alpha^* j}}\hat{\mu}_{C_{\alpha^* j}}$ and $\hat{\nu}_{V_{\alpha j}}\hat{\nu}_{C_{\alpha j}} \geq \hat{\nu}_{V_{\alpha^* j}}\hat{\nu}_{C_{\alpha^* j}}$, for all j, we have

$$\left(\left(1-\prod_{j=1}^{\hbar}(1-\hat{\mu}^{\eta}_{V_{\alpha j}})^{\xi_j}\right)^{\frac{1}{\eta}}\right).\left(\left(1-\prod_{j=1}^{\hbar}(1-\hat{\mu}^{\eta}_{C_{\alpha j}})^{\xi_j}\right)^{\frac{1}{\eta}}\right)=$$

$$\left(\left(1-\prod_{j=1}^{\hbar}(1-\hat{\mu}^{\eta}_{V_{\alpha^* j}})^{\xi_j}\right)^{\frac{1}{\eta}}\right).\left(\left(1-\prod_{j=1}^{\hbar}(1-\hat{\mu}^{\eta}_{C_{\alpha^* j}})^{\xi_j}\right)^{\frac{1}{\eta}}\right)$$

$$\left(1-\left(1-\prod_{j=1}^{\hbar}\left(1-(1-\hat{\nu}_{V_{\alpha j}})^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}}\right).\left(1-\left(1-\prod_{j=1}^{\hbar}\left(1-(1-\hat{\nu}_{C_{\alpha j}})^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}}\right)=$$

$$\left(1-\left(1-\prod_{j=1}^{\hbar}\left(1-(1-\hat{\nu}_{V_{\alpha^* j}})^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}}\right).\left(1-\left(1-\prod_{j=1}^{\hbar}\left(1-(1-\hat{\nu}_{C_{\alpha^* j}})^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}}\right)$$

Hence

$$\mathscr{H}(\alpha)=\left(\left(1-\prod_{j=1}^{\hbar}(1-\hat{\mu}^{\eta}_{V_{\alpha j}})^{\xi_j}\right)^{\frac{1}{\eta}}\right).\left(\left(1-\prod_{j=1}^{\hbar}(1-\hat{\mu}^{\eta}_{C_{\alpha j}})^{\xi_j}\right)^{\frac{1}{\eta}}\right)+$$

$$\left(1-\left(1-\prod_{j=1}^{\hbar}\left(1-(1-\hat{\nu}_{V_{\alpha j}})^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}}\right).\left(1-\left(1-\prod_{j=1}^{\hbar}\left(1-(1-\hat{\nu}_{C_{\alpha j}})^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}}\right)$$

$$=\left(\left(1-\prod_{j=1}^{\hbar}(1-\hat{\mu}^{\eta}_{V_{\alpha^* j}})^{\xi_j}\right)^{\frac{1}{\eta}}\right).\left(\left(1-\prod_{j=1}^{\hbar}(1-\hat{\mu}^{\eta}_{C_{\alpha^* j}})^{\xi_j}\right)^{\frac{1}{\eta}}\right)+$$

$$\left(1-\left(1-\prod_{j=1}^{\hbar}\left(1-(1-\hat{\nu}_{V_{\alpha^* j}})^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}}\right).\left(1-\left(1-\prod_{j=1}^{\hbar}\left(1-(1-\hat{\nu}_{C_{\alpha^* j}})^{\eta}\right)^{\xi_j}\right)^{\frac{1}{\eta}}\right)$$

$$\mathscr{H}(\alpha)=\mathscr{H}(\alpha^*)$$

Thus, it follows from Definition 2.2, that is

$$GIFCNWA(\alpha_1,\alpha_2,....\alpha_{\hbar}) = GIFCNWA(\alpha^*_1,\alpha^*_2,....\alpha^*_{\hbar}) \tag{3. 30}$$

From Equations 3. 29 and 3. 30 , we can see that the Equation 3. 20 always holds. □

The GIFCNWA operator confirms the monotonicity property by guaranteeing that the aggregated results improve as the input IFCNs improve.

4. Algorithm for decision making using IFCNs and GIFCNWA operator

The MADM is a powerful tool for addressing complex problems that involve evaluating multiple conflicting alternatives simultaneously to select the best one. In this way, it allows decision-makers to systematically trade off a large number of alternatives across multiple dimensions and to decide which criterion or alternatives are most (or least) important. In this work, we propose a MADM algorithm using IFCNs and the GIFCNWA operator. This approach, in addition to improving decision making reliability, it also enables a robust multi–criteria evaluation.

Step 1 (Attributes weights)

The weights of the attributes for each alternative can be determined using the the Analytical Hierarchy Process (AHP). This way ensures the relative importance of the attributes is effectively and objectively evaluated.

Step 2 (Input data)

Preferences from decision makers for each alternative and attribute are gathered in the form of IFCNs. Each IFCN is represented by two ordered pairs: the first pair consists of the membership grade and its credibility measure, and the second pair comprises the non-membership grade and its corresponding credibility value.

Step 3 (Aggregation using GIFCNWA operator)

In this step, we apply the GIFCNWA operator to the input data, which is collected as IFCNs. It aggregates the IFCNs of the attribute, averaged among the alternatives, with the attribute weights in the alternatives as defined by the decision-makers. To obtain a unified IFCNs, the aggregation takes into account membership and non-membership grades, as well as their credibility scores. The aggregated IFCNs are taken as the overall evaluation of each alternative based on criteria satisfaction and confidence in assessments.

Step 4 (Score function)

After aggregation, a score function is applied to the resulting IFCNs for each alternative. This function assigns a numerical value to each alternative, reflecting its overall performance across all evaluated attributes.

Step 5 (Rank of alternatives)

Using the scores derived from the score function, the alternatives are ranked in order of preference. A higher score indicates a better-performing alternative, with the highest-ranked option being the most favourable choice. Conversely, alternatives with lower scores are deemed less preferable.

The visual representation of the proposed algorithm is shown in Figure 1.

5. MADM with IFCNs and GIFCNWA operator

The MADM offers a structured approach for examining and ranking alternatives based on several conflicting attributes. Combining IFCNs and GIFCNWA operators, this method handles the uncertainties and complexity of real-world decision-making problems. In the case of solar thermal energy systems, this approach enables evaluation of alternatives with the full range of attributes using membership, non-membership and credibility measures.

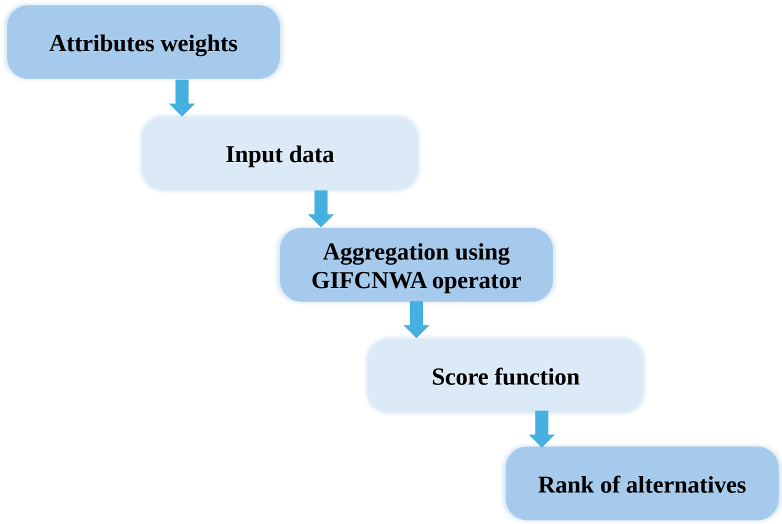

FIGURE 1. Step-by-step visual overview of the proposed algorithm

The GIFCNWA operator involves a balanced aggregation of attributes, which IFCNs deal with the vagueness in expert judgments very well. Through this advanced algorithm, we demonstrate how the challenges of thermal efficiency, material durability, and system adaptability in solar-thermal systems are systematically resolved to identify the optimal solution.

5.1. **Case study.** Solar thermal energy systems are vital components of renewable energy solutions worldwide, with tremendous potential for generating sustainable power by converting solar radiation into heat and subsequently into electricity. One of the most important functions that these systems serve within the context of global energy distribution is energy decarbonisation, in the sense of reducing dependence on fossil fuels and greenhouse gas emissions, while simultaneously supporting the transition towards a sustainable energy future. Unfortunately, solar thermal technologies have yet to live up to their promise regarding scalability and efficiency, as challenges such as thermal energy absorption, storage, and conversion hinder their improvement. For enhancing the system performance, under such real-world conditions as fluctuating solar irradiance and extreme weather, these inefficiencies need to be addressed. Several problems plague solar thermal energy systems, preventing them from being an effective technology and one that is readily adopted. The challenges encountered herein include suboptimal absorption of thermal energy due to the use of suboptimal materials, limited thermal energy storage technology, and inefficient energy conversion processes. Additionally, environmental variability and high material costs create further barriers. To address these issues, the focus shifts to selecting critical attributes that ensure efficiency, durability, and adaptability. The Table 1 below summarizes the selected attributes. Based on the selected attributes, the engineering team identified the following four alternatives $(\mathfrak{T}_1, \mathfrak{T}_2, \mathfrak{T}_3, \mathfrak{T}_4)$ as the most promising solutions to enhance solar-thermal system performance. The alternatives, along with their descriptions, are listed below in Table 2. A flowchart of the working of MADM framework is given in Figure 2.

One example is the use of Nanostructured Absorber Coatings, designed to directly increase thermal efficiency through a reduction in reflectance and an increase in solar radiation absorption, thereby reducing energy losses. Advanced Phase-Change Materials are similar, improving both thermal efficiency and material durability by providing high heat capacities for better energy storage as well as enhancing long-term durability against repeated heating and cooling cycles. For material durability, high-temperature alloys are of importance as they maintain the capability to survive at very high temperatures and pressures for applications such as jet engines and turbines, as well as heat exchangers. At last, Adaptive Control Systems address system adaptability by automatically tuning operational parameters as a function of real-time climatic conditions to ensure consistent performance across varying climatic and environmental factors. This approach explicitly couples alternatives to the attributes that they mitigate, justifying their use while providing further context on how they address fundamental problems in solar thermal energy systems. However, traditional decision-making methods such as the weighted average method (WAM) and simple additive Weighting (SAW) are found to be inadequate in dealing with the uncertain and complex nature of solar thermal system design. These methods do not account for hesitation and vagueness in expert evaluations, do not treat all inputs as equally reliable, and do not consider the interdependencies of attributes, thereby leading to oversimplified and suboptimal solutions. The team used the GIFCNWA operator to overcome these limitations. Modeling the membership, non-membership degrees of each attribute on the full spectrum of uncertainty, this innovative approach was based on IFCNs. Introducing weighted aggregation using the importance and credibility of input data, the GIFCNWA operator balanced decisions and reflected nuances in the importance and credibility of the input data. In addition, its adaptive framework was able to dynamically adjust to accommodate diverse conditions, making it highly suitable for the complex, real-world needs of solar-thermal systems. With this approach, the team was able to determine solutions that achieved the greatest thermal efficiency while also making the material and system strong and adaptable, enabling reliable and scalable solar-thermal energy technologies.

TABLE 1. Attributes with description

Attributes	**Credibility**	**Description**
Thermal Efficiency	High	Represents the system's ability to capture and retain maximum solar energy, minimizing heat losses.
Material Durability	High	Measures the resistance of materials to environmental and thermal degradation over long-term use.
System Adaptability	Medium	Ensures the system's capability to adjust dynamically to diverse climatic and operational conditions.

TABLE 2. Alternatives with description

Alternatives	Description	Background
$\mathfrak{T}_1$	Nanostructured Absorber Coatings	These coatings minimize reflectance and maximize solar absorption, improving energy capture.
$\mathfrak{T}_2$	Advanced Phase-Change Materials (PCMs)	PCMs enhance thermal storage efficiency with higher heat capacities and long-term stability.
$\mathfrak{T}_3$	High-Temperature Alloys	Advanced alloys improve the performance of heat exchangers and turbines under extreme thermal conditions.
$\mathfrak{T}_4$	Adaptive Control Systems	Real-time monitoring systems dynamically optimize operational parameters for varying climatic conditions.

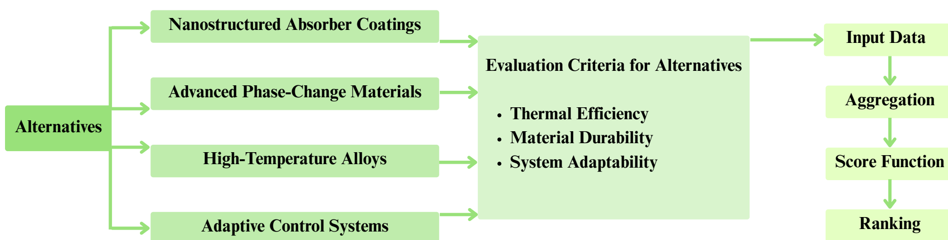

FIGURE 2. Pictorial view of the proposed MADM framework

5.2. **Numerical illustration of algorithm. (Step 1)** In this step, the AHP process has been employed to determine the weights of the attributes. A pairwise comparison matrix has been constructed to determine the relative importance of each attribute. The weights are calculated by averaging the normalized values of each row in the matrix. To ensure the reliability of the results, the consistency of the pairwise comparison matrix is verified, requiring the consistency ratio to be less than 0.1. In this analysis, the consistency ratio was calculated as 0.059682616, which meets the required threshold. Consequently, the finalized weights for the attributes are provided in Table 3.

TABLE 3. Computed weights of alternatives

Alternatives	Weights
$\mathfrak{T}_1$	0.204242342
$\mathfrak{T}_2$	0.587735711
$\mathfrak{T}_3$	0.065839299
$\mathfrak{T}_4$	0.142182648

(Step 2) The data collected from decision-makers in the form of IFCNs is presented in the Table 4. Each IFCNs is represented by two ordered pairs. The first pair indicates the membership grade and its associated credibility score, while the second pair represents the non-membership grade and its credibility score. The data collection ensured that the sum of membership and non-membership grades, as well as their credibility scores, did not exceed one.

TABLE 4. Decision maker's preferences in the form of IFCNs

Alternatives			
$\mathfrak{T}_1$	$\langle(0.1, 0.8), (0.7, 0.2)\rangle$	$\langle(0.6, 0.8), (0.3, 0.1)\rangle$	$\langle(0.7, 0.8), (0.2, 0.1)\rangle$
$\mathfrak{T}_2$	$\langle(0.8, 0.3), (0.1, 0.2)\rangle$	$\langle(0.2, 0.3), (0.2, 0.4)\rangle$	$\langle(0.2, 0.2), (0.3, 0.1)\rangle$
$\mathfrak{T}_3$	$\langle(0.9, 0.2), (0.1, 0.2)\rangle$	$\langle(0.5, 0.3), (0.1, 0.6)\rangle$	$\langle(0.4, 0.9), (0.3, 0.1)\rangle$
$\mathfrak{T}_4$	$\langle(0.7, 0.8), (0.3, 0.1)\rangle$	$\langle(0.3, 0.7), (0.4, 0.2)\rangle$	$\langle(0.2, 0.6), (0.1, 0.2)\rangle$

(Step 3) In this step, the GIFCNWA operator is utilized to combine all the IFCNs into a single aggregated IFCNs. The resulting values are presented in the Table 5.

TABLE 5. Aggregation of IFCNs using GIFCNWA operator

Alternatives	
$\mathfrak{T}_1$	$\langle(0.5154, 0.7640), (0.3814, 0.1480)\rangle$
$\mathfrak{T}_2$	$\langle(0.4579, 0.2729), (0.2087, 0.3365)\rangle$
$\mathfrak{T}_3$	$\langle(0.6367, 0.3987), (0.1379, 0.4141)\rangle$
$\mathfrak{T}_4$	$\langle(0.4215, 0.6851), (0.3683, 0.2033)\rangle$

(Step 4) At this stage, the score function (2. 1) has been applied to the aggregated IFCNs to evaluate and rank the alternatives. The computed score values for each alternative are provided in Table 6.

TABLE 6. Computed score function values

Alternatives	**Score function**
$\mathfrak{T}_1$	0.3373184
$\mathfrak{T}_2$	0.0547334
$\mathfrak{T}_3$	0.1967479
$\mathfrak{T}_4$	0.2138943

(Step 5) Finally, by applying the score function, the ranking of alternatives has been determined, as illustrated in Figure 3.

$$\mathfrak{T}_1 \succ \mathfrak{T}_4 \succ \mathfrak{T}_3 \succ \mathfrak{T}_2$$

The alternative $\mathfrak{T}_1$ (Nanostructured Absorber Coatings) is the best alternative among all the alternatives.

By considering the four alternatives and assigning weights to each, we collected the decision maker's preferences in the form of IFCNs. We aggregated these numbers into a unified IFCNs by applying the proposed GIFCNWA operator. Then, we applied the score function to aggregate IFZN in order to obtain the ranking of alternatives.

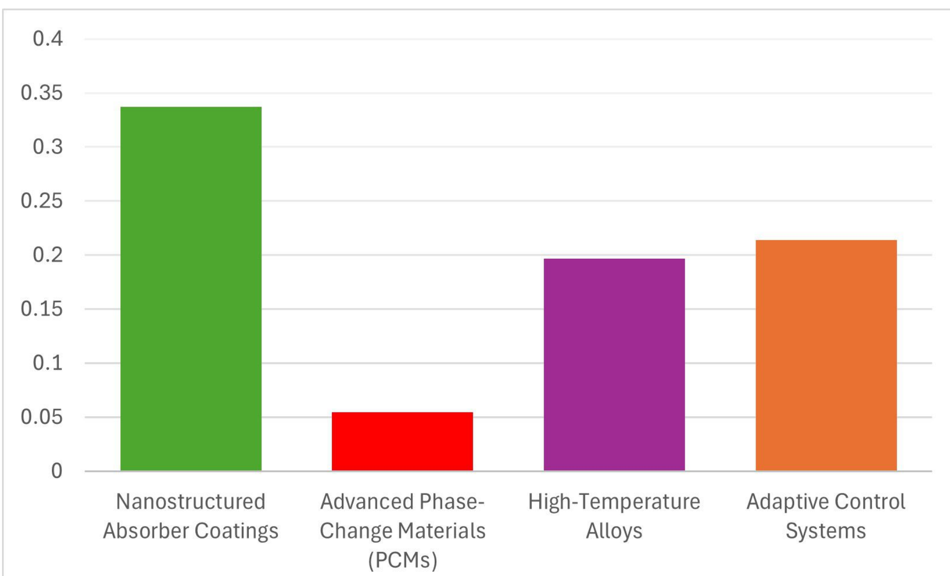

FIGURE 3. Ranking of alternatives

5.3. **Sensitivity analysis.** Sensitivity analysis is a critical step in validating the robustness and reliability of decision-making methodologies. In the proposed framework, the parameter η plays a pivotal role in the aggregation process of the GIFCNWA operator. To assess the sensitivity analysis, we performed a thorough analysis by evaluating the behavior of the proposed operator for $\eta = 1, 2, 3, 4$ and 10. We illustrate that, even under changes in η, the ranking of the alternatives is consistent. The results of the sensitivity analysis are presented in Table 7 below.

TABLE 7. Sensitivity analysis for the GIFCNWA operator with varying parameter

Parameter	**Score**				**Ranking of Alternatives**
	Score($\mathfrak{T}_1$)	**Score($\mathfrak{T}_2$)**	**Score($\mathfrak{T}_3$)**	**Score($\mathfrak{T}_4$)**	
$\eta = 1$	0.3319	0.0634	0.1939	0.2208	$\mathfrak{T}_1 \succ \mathfrak{T}_4 \succ \mathfrak{T}_3 \succ \mathfrak{T}_2$
$\eta = 2$	0.3373	0.0547	0.19675	0.2139	$\mathfrak{T}_1 \succ \mathfrak{T}_4 \succ \mathfrak{T}_3 \succ \mathfrak{T}_2$
$\eta = 3$	0.3282	0.0541	0.1938	0.2129	$\mathfrak{T}_1 \succ \mathfrak{T}_4 \succ \mathfrak{T}_3 \succ \mathfrak{T}_2$
$\eta = 5$	0.3379	0.0642	0.2021	0.2155	$\mathfrak{T}_1 \succ \mathfrak{T}_4 \succ \mathfrak{T}_3 \succ \mathfrak{T}_2$
$\eta = 10$	0.3469	0.0616	0.2013	0.2083	$\mathfrak{T}_1 \succ \mathfrak{T}_4 \succ \mathfrak{T}_3 \succ \mathfrak{T}_2$

The robustness of the the GIFCNWA operator arrives with consistency and is therefore able to efficiently provide correct results even under extremeameter conditions. The stability of such a process is crucial in MADM processes because even small changes in model parameters do not result in significant changes in the outcomes. The rankings are also invariant with respect to the value of η and hence demonstrate how the proposed operator can deal with both uncertain values and inter-dependencies between attributes. It also means that the GIFCNWA operator has an unimpaired aggregation process that does not depend on the specific value of a particular parameter within the tested range. As shown in the Figure 4, the score values vary with different parameter η, indicating the sensitivity of the GIFCNWA operator.

In particular, this is crucial for real-world applications, where the precise calibration of parameters may depend on context or decision-maker preferences.

5.4. **Comparative analysis.** Comparative analysis serves as one of the important validation methods in decision science, as it evaluates the strength, uniformity, and acceptability of a given approach by comparing it with other techniques. It confirms that the developed framework executes effectively under varying evaluative circumstances with decision stasis. In this study, a comparative analysis of the proposed MADM framework is conducted with three widely used decision-making techniques, VIKOR [29], TOPSIS [11], and WASPAS [6]. These classic approaches are well-known for their ability to execute rankings of the alternatives effectively in the context of multiple criteria. The comparative results reveal that all three methods yield the same ranking order of alternatives as the proposed GIFCNWA-based framework. The results of this analysis are presented in the subsequent Table 8.

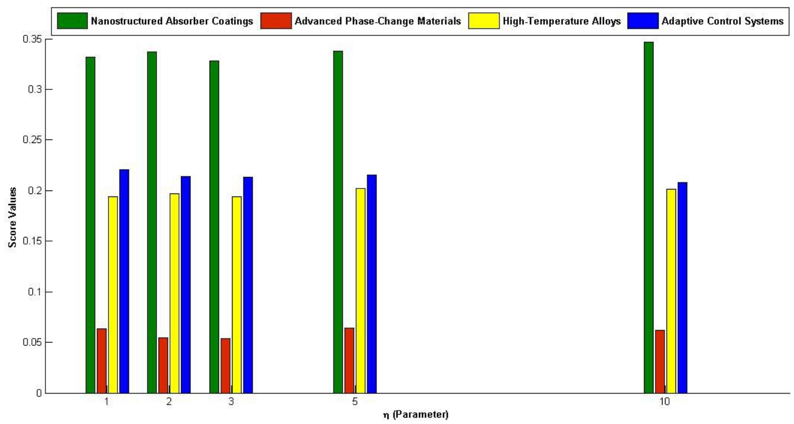

FIGURE 4. Bar graph representation of sensitivity analysis

TABLE 8. Comparison of proposed operator and traditional decision-making techniques

Methods	**Score($\mathfrak{T}_i$)**				**Ranking**
	Score($\mathfrak{T}_1$)	**Score($\mathfrak{T}_2$)**	**Score($\mathfrak{T}_3$)**	**Score($\mathfrak{T}_4$)**	
VIKOR [29]	0.0000	1.0000	0.5399	0.2035	$\mathfrak{T}_1 \succ \mathfrak{T}_4 \succ \mathfrak{T}_3 \succ \mathfrak{T}_2$
TOPSIS [11]	0.7893	0.1531	0.3455	0.7343	$\mathfrak{T}_1 \succ \mathfrak{T}_4 \succ \mathfrak{T}_3 \succ \mathfrak{T}_2$
WASPAS [6]	0.8485	0.4924	0.6575	0.7865	$\mathfrak{T}_1 \succ \mathfrak{T}_4 \succ \mathfrak{T}_3 \succ \mathfrak{T}_2$
Proposed Operator	0.3373	0.0547	0.1967	0.2139	$\mathfrak{T}_1 \succ \mathfrak{T}_4 \succ \mathfrak{T}_3 \succ \mathfrak{T}_2$

The comparative analysis presented in Table 8 demonstrates the consistency and robustness of the proposed GIFCNWA-based MADM framework when evaluated against well-established decision-making techniques. In VIKOR, alternatives are ranked based on lower score values, whereas in others, alternatives are ranked based on higher score values. All four methods yield a similar ranking order of alternatives, indicating that the proposed approach produces results aligned with conventional methodologies. This reinforces the reliability and consistency of the GIFCNWA operator in real-world decision-making problems. A pictorial comparison of these methods is shown in Figure 5.

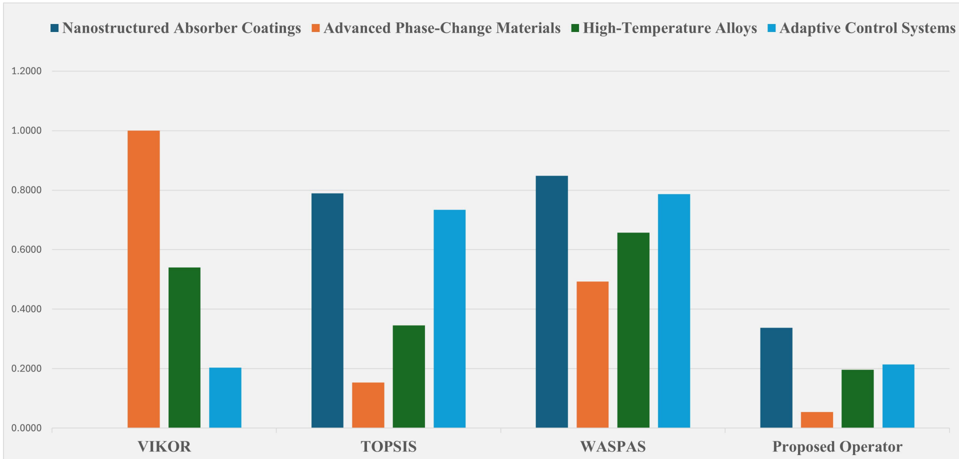

FIGURE 5. Pictorial view of comparative analysis

Traditional techniques such as VIKOR, TOPSIS, and WASPAS primarily rely on deterministic distance measures or compromise solutions that assume the data to be precise and crisp. Such models, while effective in structured environments, often fail to capture the intrinsic uncertainty, hesitation, and varying credibility that are inherent in expert evaluations. Consequently, their decision outcomes may not fully represent the nuanced judgments required in complex engineering systems. However, the proposed operator's strengths certainly go beyond the ability to generate a similar ranking. Traditional techniques primarily rely on deterministic distance measures or compromise solutions that assume the data to be precise and crisp. Such models, while effective in structured environments, often fail to capture the intrinsic uncertainty, hesitation, and varying credibility that are inherent in expert evaluations. Consequently, their decision outcomes may not fully represent the nuanced judgments required in complex engineering systems. By contrast, the GIFCNWA operator incorporates IFCNs for representing the degrees of membership, non-membership, credibility, and trust in each assessment. This aids in aggregation of uncertain information in a more clear and credible manner.

In addition to its relative consistency and interpretative foundation, the proposed framework based on the GIFCNWA operator also demonstrates computational efficiency and scalability. The algorithm primarily consists of normalizing and aggregating intuitionistic fuzzy credibility numbers, all of which operate in polynomial time with respect to the number of alternatives (m) and attributes (n). The overall calculation cost can be estimated to $O(m \times n)$, meaning linear growth with respect to the expansion of the decision matrix. The proposed operator achieves this computational efficiency in contrast to heuristic and iterative techniques, which tend to have exponential complexity. The proposed operator leverages a closed-form aggregation mechanism and a modular structure, enabling efficient computation even in high-dimensional contexts. Such efficiency not only guarantees computational feasibility, thereby allowing parallel processing of several computation steps, but also targets highly largecomplexa-driven decision environments, where both precision and efficiency are essential.

5.5. **Limitations of the study.** Although the proposed GIFCNWA-based MADM framework demonstrates notable advantages in handling uncertainty, credibility, and hesitation in expert evaluations, it has some practical drawbacks. First, the framework is dependent on the quality and consistency of expert input data. If an expert provides an assessment that is highly varied or potentially biased, credibility-weighted filtering won't completely remove subjective distortion. Second, while the algorithm works well on moderately scaled problems, real-world engineering systems will likely comprise hundreds of interdependent criteria, which will entail significant challenges with data elicitation and the real-world computational burden of evaluating large intuitionistic fuzzy matrices. Moreover, the model's static assumption on the credibility degrees can be problematic in fully dynamic decision environments operating in real-time. Finally, the current framework focuses primarily on quantitative evaluation, with limited integration of qualitative factors such as stakeholder preferences or environmental constraints, which affects its direct use in decision environments involving multiple stakeholders. Despite these limitations, the study provides a solid theoretical foundation for credibility-aware fuzzy aggregation, which can be further enhanced through hybridization with dynamic learning systems.

5.6. **Managerial implications.** The suggested decision-making process, which incorporates IFCNs and the GIFCNWA operator, has transformative implications for managerial practices in the engineering and renewable energy sectors. By tackling the intricacies of uncertainty, hesitation, and the interdependent attributes that characterize certain risks, this method equips executives with the ability to make more robust, intelligent, and data-driven decisions, even in sensitive applications such as solar thermal energy systems. A key implication is that more sophisticated decisions can be navigated with higher precision. Many traditional decision-making tools fail to capture the multifaceted interrelation of performance metrics (e.g., energy efficiency, adaptability, and system durability). By enabling expert evaluations weighted by both importance and credibility to be incorporated into decisions, this research allows managers to make decisions that are not only robust but also context-sensitive. A repeatable, structured decision-making process that removes as much of the possible subjectivity and maximises operational performance. Additionally, the incorporation of artificial intelligence in this operator using GIFCNWA operator enables this framework to be applied on dynamic and scaling system. The proposed methodology, leveraging the capacity of artificial intelligence to process large datasets and respond to dynamics evolving in real-time, is able to perform real-time evaluation and optimization of complex systems without affecting them. This adaptability can be leveraged by managers to act quickly in the face of environmental fluctuations, market demands, and technological changes, enabling organizations to remain resilient and agile with respect to their operations. In the renewable energy sector, where sustainability and cost-efficiency are paramount, this framework supports strategic planning by providing a comprehensive tool for evaluating diverse alternatives. For example, the ability to rank material choices, technology designs, or operational strategies based on multi-criteria evaluations can lead to significant cost savings, improved performance, and alignment with sustainability goals. Beyond the specific application in solar-thermal energy, this research contributes to broader managerial practices by offering a blueprint for tackling uncertainty and interdependencies in other fields, including manufacturing, logistics, and intelligent systems. By embedding AI-driven decision-making frameworks, such as GIFCNWA, organisations can future-proof their operations, ensuring consistent and optimal decision-making in an increasingly complex and uncertain world.

6. Conclusion and opportunity for future work

This research presents a novel and robust decision-making framework that integrates IFCNs with the GIFCNWA operator to address critical challenges in MADM. By overcoming the limitations of traditional methodologies, such as their inability to handle uncertainty, hesitation, and varying credibility of evaluations, our proposed approach provides a comprehensive and adaptive solution for complex, real-world problems. The reliability of the GIFCNWA operator is established through its mathematical foundation, as evidenced by the proof of key properties such as idempotency, monotonicity, and boundedness. Additionally, integrating this operator into a practical MADM algorithm enhances the precision, reliability, and scalability of decision-making processes. The proposed framework has been applied to a real-world case study involving the optimization of solar thermal energy system. Four alternatives: nanostructured absorber coatings, advanced phase-change materials, high-temperature alloys, and adaptive control systems were assessed on various criteria, including thermal efficiency, material durability, and system adaptability. Nanostructured absorber coatings was found to be the most appropriate alternative, which excelled on most criteria. To confirm the reliability of this result, a parametric sensitivity analysis was conducted, assessing the stability and consistency of the operator across different parameters. Additionally, a comparison of the proposed framework with some well-known techniques such as VIKOR, TOPSIS, and WASPAS has been conducted. The results show that the proposed GIFCNWA operator not only demonstrates ranking consistency with these techniques but also surpasses them in successfully accommodating the credibility, uncertainty, and hesitation present in expert opinions. Despite its strong performance, the proposed model has some limitations. The model depends on the value of expert opinions,, which are situation-specific, and their quality and dependability fluctuate. Moreover, with the increase in the number of attributes and alternatives, the intricacy of the resolved problems also increases.

This research provides a strong foundation for advancing decision-making frameworks and points to several promising avenues for future exploration. It can be extended to aggregate information in hybrid or fully fuzzy environments, integrating neutrosophic, hesitant, or type-2 fuzzy sets, which, which reveals the potential to work with deeper layers of uncertainty. Additionally, integrating this framework with artificial intelligence and deep learning will facilitate the creation of flexible and automated, real-time, adaptable decision systems for applications such as intelligent grid optimization, adaptive manufacturing, and energy sustainability. These would reinforce the the adaptability and robustness of the proposed operator, building the first intelligent systems for automated decision-making that address complex problems in modern engineering and energy systems.

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Conflict of interest: In accordance with ethical standards, it is essential to declare that there is no conflict of interest associated with this research article. The authors affirm that they have no financial, personal, or professional relationships that could influence or be perceived to influence the work reported in this manuscript. This commitment to transparency ensures the integrity of the research process and the unbiased dissemination of findings, fostering credibility and trust within the scholarly community.

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