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Numerical Solution of Fractional Order Epidemic Model of a Vector Born Disease by Laplace Adomian Decomposition Method

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Abstract. In this paper, we bring into focus a fractional order epidemic model of a vector -born disease with direct transmission in a population which is assumed to have a constant size over the period of the epidemic is consider. In this article, we only study the numerical solutions of the concerned model with the help of Laplace-Adomian decomposition method. We obtain the solutions of the differential equations involved in the model in the form of infinite series. The concerned series rapidly converges to its exact value. Then, we compare our results with the results obtained by Runge-kutta method in case of integer order derivative.

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Key Words: Epidemic models; Fractional Derivatives; Laplace transform; Adomian decomposition method; Analytical solution

1. Introduction

Vector -borne disease are contagious disease given birth by bacteria, viruses, protozoa which are predominantly transferred by diseases transferring biological agents known as vectors, which carry the diseases without being effected themselves. Malaria is the most predominant vector born disease whose vector are the mosquitoes. The mosquitoes are the vectors of many contagious diseases out of which dengue yellow fever , Japanese Encephalitic and West Nile Fever are the most predominant and given birth by west Nile virus. Other vectors are the assassin bugs providing the chagas disease, fleas transferring the plague from its normal hot to humans, or from human to human, and ticks which transfer the most existing vector born disease in north America [9]. Many evidence show that in the last $20-30$ years vector born disease have raised in new areas (Gulber and Marfin [12]) or re-emerged after being contained in most parts of the world except Africa in $1950's$ and $1960's$. Many factors causing the upsurge of vector borne disease have been constantly highlighted and debated in [14, 8].

The idea of mathematical modeling of spread of disease was presented for the first time D. Bernoulli in 1766, which gave birth to the start of modern epidemiology. For study of mathematical models of some physical and biological phenomenons, we refer [11, 10, 3]. In the beginning of 20th century, Ross and Hamer have also presented the modeling of infectious disease. To explain the behavior of epidemic models, they used the law of mass action. After that Reed and Frost established Reed-Frost epidemic model, which give the relationship between the susceptible, infected and immune individual in a community. Mckendrick and Kermack [7], in 1927 established a simple model known as SIR model given by the following

$$\begin{cases} \dfrac{du(t)}{dt} = -\beta u(t)v(t), \\ \dfrac{dv(t)}{dt} = \beta u(t)v(t) - \gamma v(t), \\ \dfrac{dw(t)}{dt} = \gamma v(t). \end{cases} \tag{1. 1}$$

So the total population at time t divided into three subtypes, which are susceptible, infected and recovered.

Wei et al [13], considered the system of differential equations given below, which give the dynamics of the disease in the host population

$$\begin{cases} \dfrac{ds(t)}{dt} = b - \lambda s(t)i(t) - \lambda_1 s(t)v(t) - \mu s(t), \\ \dfrac{di(t)}{dt} = \lambda s(t)i(t) + \lambda_1 s(t)v(t) - \gamma i(t) - \mu i(t), \\ \dfrac{dr(t)}{dt} = -\gamma i(t) - \mu r(t), \end{cases}$$

After some modification and by using (H. R. Thiemes [13]) the following system was presented

$$\begin{cases} \dfrac{ds(t)}{dt} = b - \lambda s(t)i(t) - \lambda_1 s(t)v(t) - \mu s(t), \\ \dfrac{di(t)}{dt} = \lambda s(t)i(t) + \lambda_1 s(t)v(t) - \gamma i(t) - \mu i(t), \\ \dfrac{dv(t)}{dt} = \lambda_2 \left(\dfrac{b_1}{\mu_1} - v(t) \right) i(t) - \mu_1 v(t), \end{cases} \tag{1. 2}$$

The above model (1. 1) was first solved by Biazar [7], by using Adomian decomposition method, Refei et al[16], by using homotopy perturbation method, Refei et al[17], by using variation iteration method, Fadi et al[5] and Abdul-Monim et al[6], by using differential transform method. The extension of this model of fractional order was studied first time in [18]. The use of fractional differential equations are basically related to biological system . Also they describe the biphasic decline behavior of infection of disease at a slow rate.

Rida et al [19], described the model (1. 1) of fractional differential equation as

$$\begin{cases} D^{\alpha_1}u(t) = -\beta u(t)v(t), \\ D^{\alpha_2}v(t) = \beta u(t)v(t) - \gamma v(t), \\ D^{\alpha_3}w(t) = \gamma v(t), \end{cases}$$

with given initial conditions

$$u(0) = U_1, v(0) = U_2, w(0) = U_3,$$

for this model the total initial conditions is $U = U_1 + U_2 + U_3$.
In the light of the above background, in this article, we bring into face the following fractional order extension of model (1. 2). In which the population of the vector is explained by a system for the vulnerable and infected vector while the dynamics of the host is explained by an SIR model. The fractional order show the realistic biphasic decline behavior of infection of disease with a slow rate. Thus the new fractional model is given by

$$\begin{cases} {}^cD^{\alpha}u(t) = \beta_1 - \lambda_1 u(t)v(t) - \lambda_2 u(t)w(t) - \mu_1 u(t), \\ {}^cD^{\alpha}v(t) = \lambda_1 u(t)v(t) + \lambda_2 u(t)w(t) - \gamma v(t) - \mu_1 v(t), \\ {}^cD^{\alpha}w(t) = \lambda_3 \left(\dfrac{\beta_1}{\mu_2} - w(t)\right) v(t) - \mu_2 w(t), \end{cases} \tag{1. 3}$$

with given initial conditions, $u(0) = N_1, v(0) = N_2, w(0) = N_3$,
where $0 < \alpha \leq 1$. In the model (1. 3), the initial conditions are independent on each other and satisfy the relation $N = u(t) + v(t) + w(t)$ where N show in the population the total number of the individuals.
For the given model of fractional order, the numerical solutions are studied by using Adomian decomposition method with Laplace transform. For the verification of our procedure, we assigned random values to the initial conditions and parameters.
In 1980, Adomian decomposition method (ADM) was introduced by Adomian, which is an effective method for finding numerical and explicit solutions of a wide and a system of differential equations representing physical problems. This method works efficiently for both initial value problems as well as for boundary value problem, for partial and ordinary differential equations, for linear and non-linear equations and also for stochastic system as well. In this method no perturbation or linearization is required. ADM has been done extensive work to provide analytical solution of nonlinear equations as well as solving fractional order differential equations. In this paper, we operate Laplace transform method, which is a powerful techniques in engineering and applied mathematics. With the help of this method we transform fractional differential equations into algebraic equations, then solved this algebraic equations by ADM.

2. Notations and Preliminaries

Here, in this section, we recall some fundamental definitions and results from the fractional calculus. For further detailed study, we refer to [4, 19, 9, 2, 15].

Definition 2.1. *The fractional integral of Riemann-Liouville type of order $\alpha \in \mathbb{R}_+$ of a function $f \in L^1([0,T],\mathbb{R})$ is defined as*

$$I^{\alpha}_{0+}f(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}f(s)\,ds.$$

Definition 2.2. *The Caputo fractional order derivative of a function f on the interval $[0,T]$ is defined by*

$${}^{c}D^{\alpha}_{0+}f(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-s)^{n-\alpha-1}f^{(n)}(s)\,ds,$$

where $n = [\alpha]+1$ and $[\alpha]$ represents the integer part of α.

Definition 2.3. *We recall the definition of Laplace transform of Caputo derivative as*

$$\mathcal{L}\{{}^{c}D^{\alpha}y(t)\} = s^{\alpha}\mathcal{L}\{y(t)\} - \sum_{k=0}^{n-1} s^{\alpha-i-1}y^{(k)}(0),\ n-1<\alpha<n, n\in N.$$

for arbitrary $c_i \in \mathbb{R}$, $\quad i = 0,1,2,\dots,n-1$, where $n = [\alpha]+1$ and $[\alpha]$ represents the integer part of α.

3. THE LAPLACE ADOMIAN DECOMPOSITION METHOD

In this section, we discuss the general procedure of the model (1. 3) with given initial conditions. Applying Laplace transform on both side of the model (1. 3) as

$$\begin{cases} \mathcal{L}\{{}^{c}D^{\alpha}u(t)\} = \mathcal{L}\{\beta_1 - \lambda_1 u(t)v(t) - \lambda_2 u(t)w(t) - \mu_1 u(t)\}, \\ \mathcal{L}\{{}^{c}D^{\alpha}v(t)\} = \mathcal{L}\{\lambda_1 u(t)v(t) + \lambda_2 u(t)w(t) - \gamma v(t) - \mu_1 v(t)\}, \\ \mathcal{L}\{{}^{c}D^{\alpha}w(t) = \mathcal{L}\{\lambda_3\left(\frac{\beta_1}{\mu_2} - w(t)\right)v(t) - \mu_2 w(t)\}, \end{cases}$$

which implies that

$$\begin{cases} s^{\alpha}\mathcal{L}\{u(t)\} - s^{\alpha-1}u(0) = \mathcal{L}\{\beta_1 - \lambda_1 u(t)v(t) - \lambda_2 u(t)w(t) - \mu_1 u(t)\}, \\ s^{\alpha}\mathcal{L}\{v(t)\} - s^{\alpha-1}v(0) = \mathcal{L}\{\lambda_1 u(t)v(t) + \lambda_2 u(t)w(t) - \gamma v(t) - \mu_1 v(t)\}, \\ s^{\alpha}\mathcal{L}\{w(t)\} - s^{\alpha-1}w(0) = \mathcal{L}\{\lambda_3\left(\frac{\beta_1}{\mu_2} - w(t)\right)v(t) - \mu_2 w(t)\}. \end{cases} \tag{3. 4}$$

Now using initial conditions and taking inverse Laplace transform in model (3. 4), we have

$$\begin{cases} u(t) = N_1 + \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}\{\beta_1 - \lambda_1 u(t)v(t) - \lambda_2 u(t)w(t) - \mu_1 u(t)\}\right], \\ v(t) = N_2 + \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}\{\lambda_1 u(t)v(t) + \lambda_2 u(t)w(t) - \gamma v(t) - \mu_1 v(t)\}\right], \\ w(t) = N_3 + \mathcal{L}^{-1}\left[\frac{1}{s^{\alpha}}\mathcal{L}\{\lambda_3\left(\frac{\beta_1}{\mu_2} - w(t)\right)v(t) - \mu_2 w(t)\}\right]. \end{cases} \tag{3. 5}$$

Assuming that the solutions, $u(t), v(t), w(t)$ in the form of infinite series given by

$$u(t) = \sum_{i=0}^{\infty} u_n(t),\ v(t) = \sum_{i=0}^{\infty} v_n(t),\ w(t) = \sum_{i=0}^{\infty} w_n(t), \tag{3. 6}$$

and the nonlinear terms involved in the model are $u(t)v(t), u(t)w(t)$, are decompose by Adomian polynomial as

$$u(t)v(t) = \sum_{i=0}^{\infty} P_n(t),\ u(t)w(t) = \sum_{i=0}^{\infty} Q_n(t),\ v(t)w(t) = \sum_{i=0}^{\infty} R_n(t), \tag{3. 7}$$

where $P_n(t),\ Q_n(t),\ R_n(t)$ are Adomian polynomials defined as

$$\begin{aligned}
P_n(t) &= \frac{1}{\Gamma(n+1)} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^{n} \lambda^k u_k(t) \sum_{k=0}^{n} \lambda^k v_k(t) \right] \Big|_{\lambda=0},\\
Q_n(t) &= \frac{1}{\Gamma(n+1)} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^{n} \lambda^k u_k(t) \sum_{k=0}^{n} \lambda^k w_k(t) \right] \Big|_{\lambda=0}\\
R_n(t) &= \frac{1}{\Gamma(n+1)} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^{n} \lambda^k v_k(t) \sum_{k=0}^{n} \lambda^k w_k(t) \right] \Big|_{\lambda=0}.
\end{aligned}$$

Using (3. 6),(3. 7) in model (3. 5), we get

$$\begin{aligned}
&\mathcal{L}(u_0) = \frac{N_1}{s},\ \mathcal{L}(v_0) = \frac{N_2}{s},\ \mathcal{L}(w_0) = \frac{N_3}{s},\\
&\mathcal{L}(u_1) = \frac{\beta_1}{s^{\alpha+1}} + \frac{-\lambda_1}{s^\alpha}\mathcal{L}(P_0) - \frac{\lambda_2}{s^\alpha}\mathcal{L}(Q_0) - \frac{\mu_1}{s^\alpha}\mathcal{L}(u_0),\\
&\mathcal{L}(v_1) = \frac{-\lambda_1}{s^\alpha}\mathcal{L}(P_0) - \frac{\lambda_2}{s^\alpha}\mathcal{L}(Q_0) - \frac{\gamma}{s^\alpha}\mathcal{L}(v_0) - \frac{\mu_1}{s^\alpha}\mathcal{L}(v_0),\\
&\mathcal{L}(w_1) = \frac{\lambda_3}{s^\alpha \mu_1}\beta_2\mathcal{L}(v_0) - \frac{\lambda_3}{s^\alpha}\mathcal{L}(R_0) - \frac{\mu_2}{s^\alpha}\mathcal{L}(w_0),\\
&\vdots\\
&\mathcal{L}(u_{n+1}) = \frac{\beta_1}{s^{\alpha+1}} - \frac{\lambda_1}{s^\alpha}\mathcal{L}(P_n) - \frac{\lambda_2}{s^\alpha}\mathcal{L}(Q_n) - \frac{\mu_1}{s^\alpha}\mathcal{L}(u_n),\\
&\mathcal{L}(v_{n+1}) = \frac{\lambda_1}{s^\alpha}\mathcal{L}(P_n) + \frac{\lambda_2}{s^\alpha}\mathcal{L}(Q_n) - \frac{\gamma}{s^\alpha}\mathcal{L}(v_n) - \frac{\mu_1}{s^\alpha}\mathcal{L}(v_n),\\
&\mathcal{L}(w_{n+1}) = \frac{\lambda_3}{\mu_1 s^\alpha}\beta_2\mathcal{L}(v_n) - \frac{\alpha}{s^\alpha}\mathcal{L}(R_n) - \frac{\lambda_2}{s^\alpha}\mathcal{L}(W_n).
\end{aligned} \tag{3. 8}$$

Taking inverse transform of (3. 8), we get the solution of u_0, v_0, w_0 , substituting these values to obtain u_1, v_1, w_1 and finally the solution in the form of infinite series as

$$\begin{aligned}
u(t) &= u_0 + u_1 + u_2 + u_3 + \dots\\
v(t) &= v_0 + u_1 + v_2 + v_3 + \dots\\
w(t) &= w_0 + w_1 + w_2 + w_3 + \dots.
\end{aligned} \tag{3. 9}$$

4. CONVERGENT OF THE METHOD

The solutions obtained in (3. 9) which are rapidly and uniformly convergent to the exact solutions. To check the convergence of the series (3. 9), we use classical techniques, (see [1]). For sufficient conditions of convergence of this method , we give the following theorem by using idea [2, 15]

Theorem 4.1. *Let $X^{'}$ and $Y^{'}$ be two Banach spaces and $A : X^{'} \to Y^{'}$ be a contractive nonlinear operator such that*
for all $x, x^{'} \in X^{'}, ||A(x) - A(x^{'})|| \leq k||x - x^{'}||, 0 < k < 1$.
Then by the use of Banach contraction principle, A has a unique point x such that $Ax = x$, where $x = (u, v, w)$. The series given in (3. 9), can be written by applying Adomian Decomposition method as:

$$x_n = Ax_{n-1}, x_{n-1} = \sum_{i=1}^{n-1} x_i, n = 1, 2, 3, \cdots ,$$

and assume that $x_0 = x_0 \in B_r(x)$ where $B_r(x) = \{x^{'} \in X^{'} : \|x^{'} - x\| < r\}$, then, we have

$$(i)\ x_n \in B_r(x);$$
$$(ii)\ \lim_{n\to\infty} x_n = x.$$

Proof. : For (i), using mathematical induction for $n = 1$, we have

$$||x_0 - x|| = ||A(x_0) - A(x)|| \leq k||x_0 - x||.$$

Let the result is true for $n - 1$, then

$$||x_0 - x|| \leq k^{n-1}||x_0 - x||,$$

we have

$$||x_n - x|| = ||A(x_{n-1}) - A(x)|| \leq k||x_{n-1} - x|| \leq k^n||x_0 - x||.$$

Hence using (i) we, have

$$||x_n - x|| \leq k^n||x_0 - x|| \leq k^n r < r,$$

which implies that $x_n \in B_r(x)$.
(ii) Since $||x_n - x|| \leq k^n||x_0 - x||$ and as $\lim_{n\to\infty} k^n = 0$.
So, we have $\lim_{n\to\infty} ||x_n - x|| = 0 \Rightarrow \lim_{n\to\infty} x_n = x$. □

5. Numerical Conclusion

Now in this section, we find numerical solutions of (3. 9) , considered the population is in equilibrium with a wild type virus, the to fond the numerical solution is in the form of infinite series by LADM, the following values are assigned to the parameters involved in the model

$$\begin{aligned} N_1 &= 3, N_2 = 4, N_3 = 5, \beta_1 = 0.1, \beta_2 = 0.5, \lambda_1 = 0.2, \\ \lambda_2 &= 0.4, \lambda_3 = 0.6, \mu_1 = 0.01, \mu_2 = 0.02, \gamma = 0.3. \end{aligned}$$

Thus using the aforesaid parameters values, we calculate the terms of the series as

$$u_0 = 3,\ v_0 = 4,\ w_0 = 5, u_1 = -8.33\frac{t^{\alpha}}{\Gamma(\alpha+1)} - 0.281\frac{t^{2\alpha}}{\Gamma(2\alpha+1)},$$

$$\begin{aligned}
v_1 &= 3.56\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 6.28\frac{t^{2\alpha}}{\Gamma(2\alpha+1)},\\
w_1 &= -10.4\frac{t^{\alpha}}{\Gamma(\alpha+1)},
\end{aligned}$$

$$\begin{aligned}
u_2 &= 0.1\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 7.7850\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} - 6.44\frac{t^{4\alpha}}{\Gamma(4\alpha+1)} - 0.4010\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}\\
&\quad - 0.125\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)} + 36.37\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\\
v_2 &= -1.257\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + 3.52\frac{t^{4\alpha}}{\Gamma(4\alpha+1)} - 0.673\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)} - 1.8\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\\
w_2 &= -3.8\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 19.08\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + 0.636\frac{t^{5\alpha+1}}{\Gamma(5\alpha+2)} + 0.987\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}\\
&\quad + 10.78\frac{t^{2\alpha}}{\Gamma(2\alpha+1)},
\end{aligned}$$

and so on.

All other terms can be similarly obtained by using the above fashion. For the analytical solution, we choose only few terms, which provide enough accurate numerical solutions. More the terms more will be accurate the solutions. In next lines, we calculate analytical solutions by taking only first three terms. Therefore the solutions after three terms become

$$\begin{cases}
u(t) = 3 - 8.23\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 7.7850\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} - 6.44\frac{t^{4\alpha}}{\Gamma(4\alpha+1)} - 0.4010\frac{t^{3\alpha+1}}{\Gamma(3\alpha+2)}\\
\qquad - 0.125\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)} + 42.65\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}\\
v(t) = 4 + 3.56\frac{t^{\alpha}}{\Gamma(\alpha+1)} - 1.257\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + 3.52\frac{t^{4\alpha}}{\Gamma(4\alpha+1)} - 0.673\frac{t^{4\alpha+1}}{\Gamma(4\alpha+2)}\\
\qquad + 5.20\frac{t^{2\alpha}}{\Gamma(2\alpha+1)},\\
w(t) = 5 - 14.20\frac{t^{\alpha}}{\Gamma(\alpha+1)} + 19.08\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + 0.636\frac{t^{5\alpha+1}}{\Gamma(5\alpha+2)} + 0.987\frac{t^{3\alpha}}{\Gamma(3\alpha+1)}\\
\qquad + 10.78\frac{t^{2\alpha}}{\Gamma(2\alpha+1)}.
\end{cases} \tag{5. 10}$$

Figure (1)-(3) shows the plot of the solutions (5. 10) obtained by using Laplace adomain decomposition method. These figures shows that the population of each class tends to decreases as the values of α_i increases for short interval of time. In addition, we given a comparison of RK4 and LAD methods in Fig. (4) for $\alpha_i = 1$ which shows that both the methods agree for short interval of time.

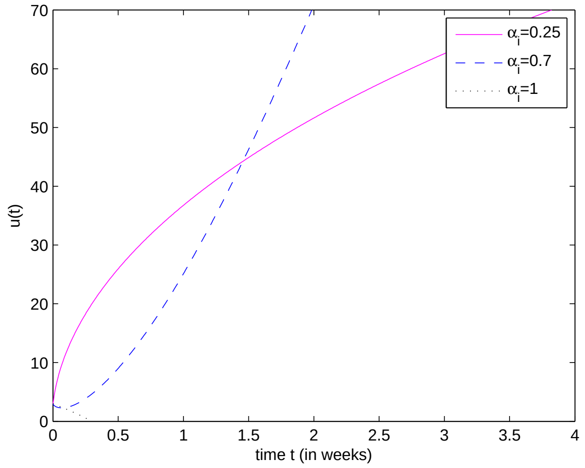

Fig. (1) The plot shows the dynamics of $u(t)$ class for different values of $\alpha_i(i = 1, 2, 3)$.

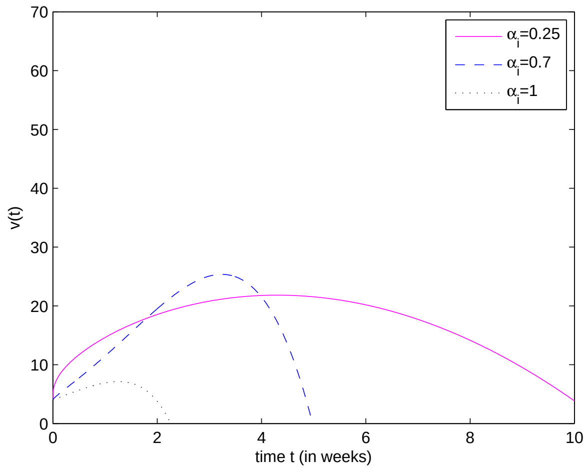

Fig. (2) The plot shows the dynamics of $v(t)$ class for different values of $\alpha_i(i = 1, 2, 3)$.

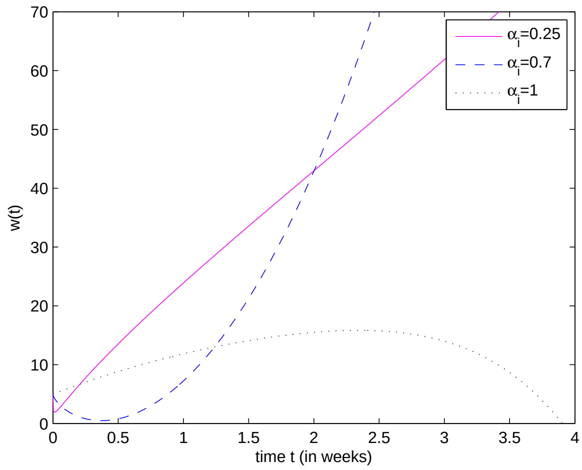

Fig. (3) The plot shows the dynamics of $w(t)$ class for different values of $\alpha_i(i = 1, 2, 3)$.

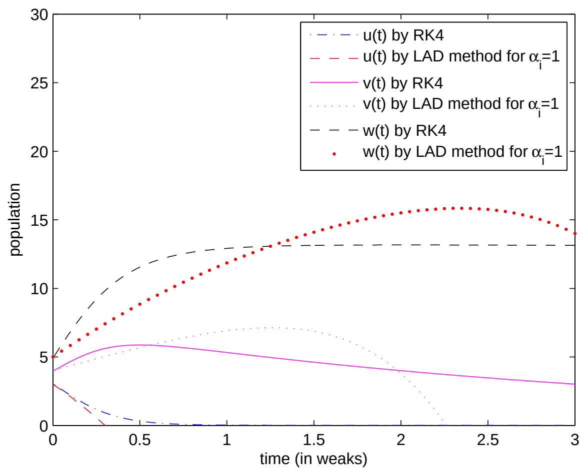

Fig. (4) The plot shows the dynamics of $u(t)$, $v(t)$ and $w(t)$ for $\alpha_i = 1(i = 1, 2, 3)$ using $RK4$ method.

6. Conclusion

In this, article, we have developed a numerical method been coupled of Laplace transform and Adomian Decomposition method. The analytical solutions have been obtained in the form of rapidly convergence series. Also the method has been compared with the Runge-Kutta method. The results have close agreement with that of the afore said method.

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